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Journal
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The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 222 AUGUST, 1936 No. 2

IDEA TO INDUSTRY.*

BY

CHARLES FRANKLIN KETTERING, D.Eng., D.Sc.,

Vice-President in Charge of Research,
General Motors Corporation.

Mr. President, Ladies and Gentlemen: It is, of course, a very great honor to receive this medal, and in accepting it I want to pay due recognition to the many co-workers who have been with me in all these various advancements. And I want to extend appreciation to a Philadelphia firm which has helped us materially—The Electric Storage Battery Company, with which we started, and which made our first batteries.

In speaking of "Idea to Industry," perhaps there is no better time to take a little consideration of such things. We have, as perhaps you may have heard, a certain unemployment problem. There are between five and ten million people out of work. That number depends very largely to which political party you belong.

We technologists have been blamed a great deal for having contributed to this unemployment. It is very peculiar how, in investigating this thing, there are so few people who appreciate that a technological development can be labor-creating to a far greater extent than it can be labor-saving. Therefore, we sometimes ask ourselves whether or not it would be better

* Presented at the Medal Meeting held Wednesday, May 20, 1936.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

if we did not have so much development, if we did not make new things which took work from other people and gave it to the machine. And yet we have very few who have ever suggested the idea that, if we extended our technological development to create new industries, we would more than absorb any people who have been thrown out of work.

I have made a study of this for the last four or five years to find out why it is that we have not extended our industrial activities. After a great deal of research, the thing that had become very, very evident indeed was that the trouble has been simply a matter of bookkeeping. We have improved bookkeeping to the extent that we do not know how to spend money without its being on the profit and loss statement. We don't know how to spend money wisely for the growth and development of a new idea.

That is quite all right, and there are no objections to a finer finesse in accounting. But we do not realize that the development from a fundamental idea to a commercial product cannot in that period be put on the profit and loss statement of any concern. That gap between the fundamental idea and the commercialized product has been widening. It has been widening for two very definite reasons.

In the first place, our technologists have become better. They have become better calculators, and therefore they do not experiment quite as far as they did in proving an experimental model. Industry has become better standardized; men have become specialized there and so they have wanted a better model than they had previously, and so for the two reasons—not having as good an experimental model—and wanting a better model—those two things have grown farther and farther apart. I drew this out for a bunch of business men the other day after the fashion of the old growth curve, the one that goes like that, and I pointed out the fact that the technical man usually stopped his activity when the growth curve started up because he felt that his idea then had enough activity to go ahead; and I also pointed out that it did not become a commercial thing until the product began to make a profit. That zone in between the idea and the finished product I called, for want of a better name, the "shirt-losing zone." It is in that place where, if there is not proper manage-

ment, proper engineering and proper study, a great deal of money can be spent, some very bad products can be manufactured, and some very grave losses sustained.

It is the fear of that zone that makes the manufacturer want to move away from it, and it is the fact that the technical man does not want to be reduced to commercial point of view that makes him back away from it.

I have been trying to work out for a great many years a method whereby I could serve as a peace maker, so to speak, between these two groups. I tried to analyze why it was that our acounting got us into trouble. We have learned how to specifically allocate to every piece and operation its own costs and its own work. That represents ninety-nine and a fraction per cent. of the total business of a concern. And then, if you set up a so-called industrial research laboratory alongside of that, it is hard to get the accounting system and the management to recognize that the money spent in an industrial research laboratory cannot be specifically allocated to a particular operation or a particular piece. And, therefore, the accounting system naturally doesn't like the rather loose way in which a research laboratory has to be run.

But there is another type of accounting which is well known in the city of Philadelphia, and that is what I term actuarial accounting—the insurance way of accounting—in which you are not so particularly interested in the individual thing involved as you are over the grand average of a lot of figures. But it is hard to understand that a research laboratory is nothing but an insurance company for the particular industry in which it works; and, therefore, the appropriation which is given to the maintenance of a research laboratory should be regarded as an insurance policy against two things, against ignorance of what is going on in the world, and against not recognizing that we must make profits.

So in accepting our responsibility we say that we will take the appropriation, whatever amount they care to pay, for that insurance policy, and we will really spend it with the same degree of carefulness and thrift as we spent for manufacturing; but we cannot guarantee when, what or why we will produce income, and naturally you can see how that does not fit into the specific allocation to a great many people's minds. It's a

lack of understanding of how much work there is to take a thing from an idea to a commercial product. That is one of the reasons why industry is not developing in this country the way it should. We need to know how to take the idea to bring it into at least a reasonable commercial form and present it to the public for their acceptance or rejection.

There is another great difficulty in the idea development process and that is in trying to attempt to lay out the road or the course by which an experiment should go. You cannot do that, and, therefore, any committee that you have that is willing to sit in on the procedure of any research development in the world will throw it off the track because they want to prescribe in advance the method of procedure of traveling over a road over which nobody has hitherto gone. Therefore, when you reach a point where you'd like to change direction, they say the committee has agreed that we will do so and so, and it can't be done that way. We know so little about everything that it's a shame that we have to stand back and work with these fundamental psychological problems.

One of our medals here received is awarded for papers written on the subject of lubrication. And lubrication brings along with it its fundamental need or cause and that is the question of friction. I am talking about physical friction now. And yet we wipe our hands together and they become warm and you say: "Why did our hands get warm?" They say it is on account of the friction. We ask them what the definition for friction is, and they will tell us it is the thing that causes your hands to get warm when you rub them together. The mechanism of friction is the absolutely unknown, or at least I haven't been able to find with our group of men what they are willing to write down as the specific mechanism of many of the most fundamental things in mechanical engineering friction. Well, the reason is not a very spectacular thing and so we go from that to any other thing you want to go to as to the question of hardness, or the question of metallurgical rearrangements of particles of metal. We have a lot of words for these things, and the longer the words are the more we think we know about them, and so it's difficult then to get back, to get men enthusiastic over doing those very elementary things which have to be done in order to bring the idea into the form

in which the public can use it and for which they are willing to pay money to buy.

You can take that all through the line in the subject of electricity—the question of magnetism—the question of electrostatic force—old, die-covered before the present Christian Era. Yet in reading a book the other day I noticed that the gentleman who wrote it said that it is a very interesting thing to note that in these great phenomena which we discovered around six or seven hundred years B.C. we know little more about them now than we did then, so far as their fundamental nature is concerned.

So we have to go back again to some of the very basic principles if we are going to expand industry.

In the particular industry which I represent, we have three very important factors that have made it successful. One has been rubber. One has been petroleum products. And one has been alloy steel. In those different industries almost all the great steps have been made by somebody discovering that some of the rules and regulations which had been laid down governing that particular subject didn't follow beyond a certain projected point. Nobody would have thought that a rubber tire would have been the best kind of a tire to run on hard ground or the irregular roadway. It is interesting to note that the inventor of the pneumatic tire, Mr. Dunlap, wasn't thinking of it in that term at all. He undertook to provide a better tire for his son's bicycle. It had hard tires, and he ran it over the rough cobblestones. Mr. Dunlap took an ordinary board and sawed it out in a size comparable to that of a bicycle wheel. He put canvas over it, and inside inserted a rubber tube blown up with a football pump. That was the beginning of the pneumatic tire. That is quite elementary to what has been done on our modern pneumatic tire, yet the steps that followed from that simple elementary tire to these wonderful things we have today is a revelation of the breaking of straight line concepts.

Everybody said it was impossible to have that, and finally an unknown bicycle racer beat a champion for the simple reason that he had pneumatic tires. Then the thing was taken up with engineers, who saw all of the complications that looked as though they could not be surmounted at all. After

they all failed, one by one, we were willing to let the problem be the boss instead of the boss selecting the problem. That is one of the most fundamental things in all engineering. The problem must be the boss, and it does not make any difference what you think about it, it is going to keep on being the same problem it is, regardless of what you think.

In analyzing the great possibilities ahead of us in industry, and knowing what we know, and what we don't know, I have suggested to a bunch of people that if we could get an analysis or rather an outline of what we don't know, what a marvelous thing it would be. We look at what we know, and we pride ourselves on what we know, but it doesn't give us a comparison. If we just had an outline—here's what we know; over here's what we don't know—then we would see the possibilities ahead of this country from an industrial standpoint.

Our recent depression has not been a depression because of lack of basic material, lack of engineering ability or anything else; we simply got into a mental rut due to looking at statements rather than necessities. We arrived at a point where we stopped. A great many people said: "There isn't much to be done any more." There isn't much to be done any more if we look at the thing as a finished book, but if we look at the thing as only a primary chapter, there is a certainty that American industry is just beginning.

I thank you very much.

TELEPHONY—CHILD OF ORGANIZED SCIENTIFIC RESEARCH.*

BY

FRANK B. JEWETT, Ph.D., D.Sc., D.Eng., LL.D.,

President and Director, Bell Telephone Laboratories, Inc.

In accepting its Franklin Medal with the Certificate of Honorary Membership which accompanies it, and in expressing to The Franklin Institute my profound appreciation of the honors thus conferred, I am not unmindful of the fact that in reality they are honors to my associates more even than to me.

If during the past three decades I have been able to contribute something to the science and art of telephony in particular and to electrical communication in general, it has been not primarily anything of outstanding technical importance. Rather it has been aid in the direction of enabling many men to develop the maximum of their creative scientific ability; of providing and maintaining a proper atmosphere for cooperative endeavor, which is the sine quo non of success in modern industrial research, and of applying critically to results and proposals the tests which come with age and much sad experience.

From such a background of realization you will understand why it is that I shall always consider these honors as things acquired in trust. It is easy for a superior to take or appear to take credit for the accomplishments of younger associates—sometimes it is difficult to avoid doing it. What is hard is regularly to appear in the role of a critic—an apparent reactionary in the ranks of ardent enthusiasts. Sometimes I become terribly tired of my part. However, as I look back over more than thirty years of work in this field of applied science I have a feeling that I may have contributed as much toward advancing the art of electrical communication by what I have prevented being done as in what I have encouraged.

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* Presented at the Medal Meeting held Wednesday, May 20, 1936.

Telephony, and in fact all forms of electrical communication, is truly a child of organized scientific research. It was born in a laboratory, it developed in a laboratory and with the passing years it came more and more to derive its life blood from the creations of skilled scientists associated together cooperatively in great industrial research organizations.

The fact that telephony in our modern world has come to be an extensive and vital part of our social and business structure tends sometimes to mask the dependence of the service on the research men. The huge sums of money invested in plant; the thousands of employees and the necessity of providing for their wages, their comfort and their continued esprit de corps; the millions of daily contacts with customers; the vast and intricate relationships with government and a host of other things, all of vital importance which each day have to be properly handled, create a superstructure which makes it easy to lose sight of the fact that, one and all, they are supported by the things which scientific men in laboratories have produced. Without these they would not exist. Further, so long as there is scientific knowledge, existing or acquirable, which can still be applied to better, to cheapen and extend existing modes of electrical communication or to create new ones, the whole superstructure is subject to realignment as a result of what these laboratory men do. Nor can we stop the tide if we would unless perchance we are prepared to commit social hara-kiri.

Limitation of time compels me to speak of but a single child of organized scientific research—telephony. What is telephony? Technically it is all forms of electrical transmission of intelligence in which the ear is the ultimate detector and translator to the brain. Telegraphy, on the other hand, is electrical transmission of intelligence in such manner that the eye is the ultimate detector and translator. The instruments at the sending and receiving end of the circuit, the type of transmitting circuit and all the intermediate apparatus may vary greatly without altering the fundamental concept of what constitutes telephony. All that we commonly think of as telephony, all radio broadcast and all the sound portion of talking pictures is telephony by this definition. They are so treated in the research laboratory since they require mere

variant applications of a common knowledge and a common technique. Usually solution of a specific problem in one sector solves or helps to solve problems in the other sectors.

A brief look backward will show clearly why telephony in all parts is so completely and necessarily a child of organized scientific research. When Dr. Bell made his discovery it was a scientific achievement involving the results of scientific discovery by others. It was not, however, organized research in our present-day sense. Nor was the work which followed in the years immediately succeeding the Bell invention. It was scientific and much of it was done under truly laboratory conditions but in the main it was the work of scientists, engineers, experimenters and inventors working more or less as independent units. This was a necessary condition and one always present in any new art. Everything had to be developed, facts of a simple character had to be obtained, men had to be educated and, above all, underlying science had been explored only in surface spots.

For a few years there was much to do in developing more efficient and rugged transmitters and receivers, in evolving satisfactory signaling devices and in working out switching arrangements for the expeditions connecting together of subscribers' lines. All of these things could be done by ingenious men familiar with and borrowing from the older telegraph art. In these years commercial telephony was wholly a community affair in which the transmission efficiency of simple line structures more than sufficed to provide for the inefficient terminal apparatus possibilities. Nor were there in these local systems many problems of interference either between telephone lines themselves or from other electric circuits. The lines were so few and so scattered that the simple expedient of physical separation usually sufficed when occasional trouble occurred. Further there were no particular problems of space or cost in the central offices arising out of great congestion of line terminals.

Gradually, however, as instruments were improved in efficiency and reliability and as the use of this new means of distant communication increased, the need for and possibility of intercommunity service developed. With it arose the beginnings of those transmission problems and the problems of

vast and intricate switching offices which have been our main concern for four decades.

At first the transmission problems were overcome by using larger wires, by a radical departure from the ground return circuit art of telegraphy to an all metallic circuit and by a substitution of high conductivity hard-drawn copper wire for the iron or steel wire previously employed.

In the central offices, application of ingenuity in the reduction of part sizes and their more orderly arrangement combined with clever circuit arrangements sufficed for a time.

Soon, however, it became increasingly apparent that the obvious and easily followed roads of advancement were becoming fewer, narrower and progressively more expensive. Likewise, the limitations of a purely individual type of trial and error attack on the numerous problems pressing for solution became manifest. Were the obvious economic and social advantages of a better, cheaper and more universal development of telephony destined to be unduly hampered if not blocked by the difficulties immediately ahead?

Fortunately for telephony and for many other arts as well, the two decades from 1880 to 1900 were extremely prolific years in the fields of fundamental science research and in the rapid adaptation of educational curricula to the continually mounting store of new knowledge. A survey of telephony's problems in the light of what had happened in these two sectors convinced us that the problems were not insoluable. If, however, the solutions lay hidden in this newly acquired and rapidly growing fund of knowledge and in the techniques which had evolved it, it was equally apparent that they could only be had through the introduction of those same techniques and of men trained in them and with an understanding of the facts already unearthed.

Nor was the indicated course merely one simply of introducing men and techniques and proceeding along the essentially individualistic lines of the established development art. Most of the techniques as well as most of the new knowledge had been evolved by scattered individual effort in institutions of learning. Rarely, if at all, had the problems solved involved the necessity of simultaneous solution of a host of other problems in widely scattered fields; rarely was time, so

vitally important in industry, an essential essence of the undertaking.

If, therefore, our salvation lay in the direction indicated, its attainment required a radical step forward in the use of science and scientific men. Moreover, it could not mean a complete break with all the values of the past. It must be a gradual introduction designed to refructify these old values, to indoctrinate the older order of men already successful in an earlier battle with a sense of the power inherent in the new tools and so ultimately to reorganize the forward movement.

Was it possible to organize scientists for a mass attack on problems which might involve almost anything in the whole domain of the physical sciences? Could we analyze these problems in sufficient detail to permit of attacking the several parts successfully and on time? Could we synthesize our solutions in a form to make them commercially available? Would fundamental science continue to produce new knowledge in sufficient amount to justify the grand adventure? These and a hundred other similar questions confronted us and had to be answered affirmatively very largely on faith.

From the vantage point of our present knowledge the questions seem trivial and the answers obvious. Thirty-five years ago it was otherwise. Then industrial research in any field as we now know it was sub-embryonic. We had a little experience and none at all in fields where the problems were so vast or the stakes of success or failure so great.

The story of the past three decades is far too long even to be touched on here. To a very large extent it is the story of the research department of the Bell System—the story of an organization which in that time grew from a handful of men to the present Bell Telephone Laboratories with its staff of approximately 4,500 men and women. What can be enumerated briefly are some of the more important direct results of organized scientific research in the field of telephony. They are results which in the main could not have been obtained without it.

It has given us our vast urban telephone networks without unsightly pole line structures; it has made not two blades of grass grow where one grew before, but literally scores of pairs of wires grow in cables in the same space where one grew

before and has provided the means which make these present fragile filaments of copper more efficient conductors of speech than their much more robust ancestors. It has greatly improved the efficiency, uniformity, reliability and convenience of the instruments we use in our homes and offices. It has revolutionized the art of switching until our more recent central offices bear scant resemblance to those of even the World War period. It has enabled us to extend the range of commercial telephony from less than 1,000 miles until there are no land distances over which speech of adequate intensity and quality cannot be given. At the same time it has so altered, cheapened and improved the transmission circuits that today we can afford to have enough of them of proper character to permit of completing practically every connection asked for while the calling party remains at the telephone.

It has developed radio telephony so that ocean barriers to speech have been destroyed, and likewise communication to and between ships, airplanes and all moving vehicles made possible. It has developed radio broadcast and made possible a great new industry in the realm of disseminating news, music and entertainment. It has made possible the simultaneous transmission of a host of non-interfering messages over the simple circuit which before its advent carried but a single message and that but poorly. Finally, it has made possible the interconnection of all these things in any random fashion and in addition has given birth to a great new industry where sight and sound are joined for our edification and education.

Many of our present-day services of telephony could never have been done at all without the aid of organized scientific research. None of them could have been accomplished so completely and cheaply had we had to depend on the fortuitous outcome of unorganized endeavor no matter how scientific its elements. The history of each success in every major and minor achievement of telephony's organized research is generically the same—analysis, concentrated attack on the elementary problems and synthesis to a final answer. Every branch of mathematics and of the physical sciences as well as many of the biological sciences has been examined and drawn on for aid; every new piece of knowledge which might be useful

has been scrutinized; new knowledge in the fundamental sciences has been sought to the end that a better, cheaper and more nearly universal telephone service might be afforded the people of the United States. As an example of how far afield the quest has taken us might be mentioned the fact that some years ago it became apparent that the whole structure of sound scientific research in many branches of telephony was based on having a much better quantitative knowledge of the exact mechanism of speech and hearing than then existed. We had then to adventure into the realms of physiology and anatomy. We have obtained much of what we sought. Incidentally we have, I think, enriched those fields.

So much for the present. What of the future of organized scientific research in the field of telephony? Has the day of its zenith been reached? Have so many frontiers been crossed in the past thirty years that there are few that remain to be conquered? What is the future of this corps of highly trained scientists?

It is true in electrical communication as it is in every other field that a frontier once crossed can never again be a frontier. To those of us for whom the spanning of a continent or an ocean with speech channels was once a challenge, our success closed the door forever to later generations. Today there is no barrier of distance or in the character of terminal station to telephony anywhere on the globe where man may be. The establishment of telephone channels is purely a matter of economics or of policy. But does this mean that in the future there is little place for an agency which for three decades has demonstrated its creative power in ever increasing measure? I think not, although it is obvious that the character of many of the objectives will be different from those of the past.

So long as efficiency and cost are factors limiting the free and untrammeled use of telephony, or so long as there is hazard to its operation through the development of interfering services, there will continue to be a challenge to organized scientific research. New increments of knowledge in many fields of science are potential tools in the field of a more efficient telephony. They may be knowledge as to new materials, new discoveries in mathematics or new concepts of atomic structure, but they cannot be neglected and a conquest of

their possibilities will be as intriguing and profitable as any of the grosser adventures we older men have engaged in.

Even on the grosser side the future still presents the challenge of how best to render *all* the services of electrical communication. We have already gone far enough, as a result of our research work, to know that from a technical standpoint all of these services are part of a common science. We have still, however, to work out, partly in the field of science and engineering, partly in that of economics, and partly in that of government, the best way of handling the problem. The final answer will be governed very largely by what organized scientific research has to say on the subject.

We have built up a great organization of scientifically trained men competent to contribute new knowledge of a fundamental character, to attack expeditiously and efficiently any problem which presents itself, and with a demonstrated capacity to produce results of a value far exceeding the expenditure of time, effort and money required in their production.

Without venturing on the unsafe ground of speculation as to what further research in fundamental science may produce, we nevertheless know that there are still great vistas of possibility for scientific research in the fields of telephony and its sister services. It is still true here, as it has been for many years past, that each advance opens the way to yet other advances. For example, in the field of multiple transmission of messages over relatively cheap and simple conductors, we know that it is theoretically possible to go infinitely farther than we are yet prepared to venture. How far it will be commercially practical to go is something for the future to determine. In the last analysis it is a problem for meticulous organized scientific research, to which finally the cold impersonal judgment of economics is applied.

In other words, modern telephony, which is surely the child of organized scientific research, seems to me destined to continue in this status for many years to come.

THE FORCES ACTING ON HIGH ENERGY PROTONS AT COLLISION.

BY

ARTHUR BRAMLEY,

The Bartol Research Foundation of the Franklin Institute.

ABSTRACT.

BARTOL RESEARCH FOUNDATION
Communication No. 93.

The interaction between protons at close collisions has been investigated on the basis of the Born-Infeld theory of electrodynamics. The theory predicts, first, that the interaction should have a definite upper limit which depends on the value of the characteristic constant b; secondly, that the magnetic field set up by the moving charges can produce in this interaction, in addition to the terms required by the classical theory of electrodynamics, terms which cannot be neglected in the treatment of close encounters. These terms become of special significance in that problem whenever the relative velocity of the two protons exceeds 3% of the value of the velocity of light. The bearing of these conclusions on the experiments on the scattering of protons by protons is discussed.

INTRODUCTION.

Recent experiments on the scattering of beta particles by atomic nuclei and on the behavior of protons at collision with protons have raised the question of the nature of the interaction between high energy particles. The classical concept of this interaction does not seem to be adequate for the explanation of the observations. In a closely related set of phenomena, the action of the primary electrons constituting part of cosmic radiation, it has been necessary to revise the classical notions concerning the interaction of very high energy electrons with matter in order to obtain theoretical results more in harmony with the experimental results. A set of criteria for the validity of the applications of electrodynamics to these problems has been devised by Oppenheimer.[1] A justification for these criteria can be found in the modification of the classical equations of motion introduced by Dr. W. F. G. Swann[2] or in the system of electrodynamics fromu-

[1] J. R. Oppenheimer, *Phys. Rev.*, **47**, 44, 1935.
[2] W. F. G. Swann, *Phys. Rev.*, **49**, 829, 1936.

lated recently by Born and Infeld.[3] Since the results of these theories have been so successful in their application to that problem it seems desirable to investigate thoroughly the form of the interaction of relatively slow particles on a corresponding basis. As the mathematics of the Born and Infeld theory are more suitable for the purpose at hand, we shall study in this paper, what effect the magnetic field can produce on such an interaction on the basis of this theory. This point is not superflous because in a non-linear field theory a magnetic field can contribute to the force acting on a stationary particle.

CONSEQUENCES OF THE BORN-INFELD THEORY.

For the sake of simplicity, the particle of charge Ze and mass M will be taken at rest at the origin of the coordinate system. The problem that we wish to investigate is the influence which the field of another particle of mass M' and charge Ze' has on this stationary particle. If the latter particle is moving along the direction of the x axis with a velocity v, then, on classical dynamics the force experienced by the stationary particle is given by ZeE where E is the electric intensity component of the field of the moving particle. The field representing the electric intensity has the value $Z'e/r^2 \left(\frac{x}{r}, \frac{\gamma y}{r}, \frac{\gamma}{r} \right)$, where γ is the quantity $1/\sqrt{1 - v^2/c^2}$ and r is the distance between the two particles. On the classical theory, the magnetic field of the moving charge has no influence on the force exerted on the stationary charge. This is not true, however, in a non linear field theory such as the Born unitarian field theory. We shall proceed now to develop the possibilities of the theory in this connection.

The Born theory of electrodynamics makes a distinction between the vector D—the electric induction—which is a solution of the equation div $D = 0$ in the absence of charges, and the electric intensity E, in terms of which the force on a stationary charged particle is expressed.

A similar distinction is made between the vector H—the magnetic intensity—which is the solution in a space free of

[3] M. Born and L. Infeld, *Proc. Roy. Soc.*, **144**, 425, 1934.

magnetic poles of the equation div $H = 0$ and the magnetic induction B.

In order to specify completely the field in free space it is necessary to add to the two sets of Maxwells' equations

$$\begin{aligned} \text{rot } E + 1/c \frac{\partial H}{\partial t} &= 0, \qquad \text{div } H = 0, \\ \text{rot } B - 1/c \frac{\partial D}{\partial t} &= 0, \qquad \text{div } D = 0, \end{aligned} \tag{1}$$

two additional relations between the inductions and the intensities. These relations are supplied in the Born electrodynamics by the following equations

$$B = \frac{H - GE}{\sqrt{1 + F - G^2}}, \qquad D = \frac{E + GH}{\sqrt{1 + F - G^2}}, \tag{2}$$

where F and G are the invariants

$$F = H^2 - E^2, \qquad G = (H \cdot E).$$

The fields in these equations are measured in terms of the natural unit of electric intensity. According to Born and Infeld this unit b is equal to 9.2×10^{15} e.s.u.

Equations (2) correspond closely to the additional equations required for the specification of the properties of a ponderable medium. In that case they relate the induction to the corresponding intensity through a linear relationship whose proportionality factors are respectively the dielectric constant and the magnetic permeability.

The charges are introduced into this scheme of electrodynamics by the method devised by Larmor. The charges are defined as existing at those points at which there exists a singularity in D. Simultaneously the equation div $D = 0$ becomes no longer satisfied.

Let us consider now the forces acting on a charged particle in an external field; and, in particular the forces acting on a particle at rest at the origin which arise from the moving charge of mass M'. The values of the electric field E'' which result from the presence of the moving particle can be easily derived from the values of the electric intensity E' of the moving charge in that system of coordinates in which it itself

is at rest with the help of the relativity transformation. These values are

$$E_x'' = E_x', \qquad E_y'' = \gamma E_y', \qquad E_z'' = \gamma E_z'$$

$$\text{and} \quad H_x = 0, \qquad H_y = \gamma \frac{v}{c} E_z', \qquad H_z = -\gamma \frac{v}{c} E_y', \qquad (3)$$

where $$\gamma = (1 - v^2/c^2)^{-\frac{1}{2}}.$$

In the Born electrodynamics the separation of the total field into two parts—that arising from the charge under consideration and another part arising from the field of the external charge—is not possible as the components of E and H appropriate as solutions for the separate entities are not additive to give a solution appropriate as the simultaneous solution of both entities. It has been remarked by Frenkel,[4] however, that the law of additivity holds approximately for the induction vectors D and H since they are connected with the true distribution of charge and current density by exactly the same equations as hold in the Maxwellian theory. As Frenkel further points out, the additivity law is certainly accurate when one of the induction vectors D, for example, is such that $D'' \backsimeq E''$. For the problem in which we are interested viz. that of two charges, one fixed and the other moving, the problem of the additivity of the magnetic induction need not concern us as long as we can neglect the intrinsic magnetic fields of the charges themselves. Since $D'' = E''/\sqrt{1 - (E')^2}$, the condition $D'' \backsimeq E''$ does not require that $E'' = (\gamma E') \ll 1$ but that $E' \ll 1$. This restriction $E' \ll 1$ is always satisfied for Born's value of b when the charges are at a greater distance than 5×10^{-13} cm. from one another.

For the derivation of the interaction between the two particles, we shall adopt the method devised by Frenkel for the treatment of this problem. As he points out, the really important quantity is the energy tensor of the electromagnetic field. The equations of motion are then derived from the principle of its conservation.

Let us construct now the total electromagnetic energy density function U of the particle at the origin, together with

[4] J. Frenkel, *Proc. Roy. Soc.*, **146**, 930, 1934.

the contribution, in that region, of the electromagnetic field of the remote particle. In order to construct this function U it is necessary to compute the values of the inductions. As we have already seen the total electric induction D can be expressed in the form $D = D_1 + D''$ i.e. as the sum of the two components $D_1 = E_1/\sqrt{1 - (E_1)^2}$ and $D'' = E''$. Here E_1 represents the electric intensity produced by the particle at the origin considered as an isolated system. Neglecting the contributions to the total energy density arising from the magnetic fields alone, since they play no role in this discussion, the energy density of the system in the neighborhood of the origin can be expressed in the form $U = (\sqrt{1 + (D^2 - B^2) - (D \cdot B)^2} - 1)$. The energy density of the field of the particle at the origin considered as an isolated system is $U_1 = (\sqrt{1 + D_1^2} - 1)$.

If now we consider only that region where $D_1 > 1 \geq D''$ and if we retain only the first term of the expansion, then the energy density can be written in the form

$$U = \left\{ (\sqrt{1 + D_1^2 - (D_1 \cdot B)^2}) \times \left(1 + \frac{2\, D_1 \cdot D'' + (D'')^2 - B^2}{2\,(1 + D_1^2 - (D_1 \cdot B)^2)} \right) - 1 \right\}$$

$$\leq \left\{ (\sqrt{1 + D_1^2 - (D_1 \cdot B)^2}) \times \left(1 + \frac{D_1 \cdot E''}{1 + D_1^2 - (D_1 \cdot B)^2} \right) - 1 \right\}. \qquad (4)$$

In the case where $B = 0$, this expression reduces to the one that we would deduce from classical considerations. The term $U_{11} = \int (\sqrt{1 + D_1^2} - 1) dv$, where the integration is carried out over the volume of the particle at the origin, gives exactly the electrostatic energy of that particle. The last term $U_{12} = \int E'' \cdot D_1/\sqrt{1 + D_1^2}\, dv$ is equal to $\int E'' \cdot E_1\, dv$, and is the mutual potential energy of the two charges. However in the case where B exists the expression for the mutual potential energy is changed by the incorporation of the term $-(D_1 \cdot B)^2$ into the denominator of that expression so that now $U_{12} = \int (E'' D_1/\sqrt{1 + D_1^2 - (D_1 \cdot B)^2}\, dv$. This factor

added to the denominator has considerable influence on the value of U_{12} whenever B becomes of the order of unity in the natural system of units, i.e. when $\gamma \frac{v}{c} E' \sim 1$.

A discussion of the circumstances under which this condition is satisfied for the proton-proton interaction will be given latter on.

The expression for U_{12} shows that the effect of the additional term in the denominator is to increase the mutual electrostatic energy of the system of two charges. Translated into classical dynamics, this is equivalent to increasing the value of the apparent electrostatic force acting on the stationary charge. Thus in contradistinction to Maxwellian electrodynamics, the Born unitarian field theory predicts that the interaction should be enhanced by the presence of the magnetic field to an extent greater than that predicted from the application of the Lorentz force equation to this problem. The special feature is that the added interaction is finite even when the velocity of the particle at the origin is zero.

Two distinct features are recognized at once from a consideration of the implications involved in the condition $\gamma \frac{v}{c} E' \sim 1$. We notice first that the value of E' depends on the position of the charges producing it. Secondly that the value of E' for which the condition is satisfied depends on $\gamma \frac{v}{c}$, in which the velocity is measured relative to the particle at the origin, and on the value chosen as the basis of the natural system of units.

At the present time the most interesting case to consider is that of the scattering of protons by protons. There, considerable deviations from the results calculated from that form of the classical theory which is applicable to point charges, has been observed for those collisions in which the impact parameter is less than 6×10^{-13} cm. For these collisions the relative velocity is greater than 0.03 of the velocity of light. If the influence of the magnetic field on the electrostatic interaction is the source of part of this discrepancy then the above calculations require that the base b of the natural

system of units must be of the order of 10^{14} e.s.u. since $\frac{v}{c} \sim 0.03$ and $E' \sim 10^{15}$ e.s.u.

The exact value for b for a proton is unknown. For the electron, Born and Infeld have calculated from electrodynamical considerations the value 9.2×10^{15} e.s.u. On the other hand, Heisenberg and Euler [5] have computed from their development of the Dirac-Heisenberg picture of the negative energy states the value $9.2 \times 10^{15}/137 = 0.7 \times 10^{14}$ e.s.u. If either of these formula for the numerical value of b were valid for other values of the mass besides that of the electron, then, an extrapolation to the value for the mass equal to that of the proton would yield for b a value $(1836)^2$ times that for the electron. It has been remarked by Born [6] and by Schroedinger that such a value for b is open to the very serious objection that, in calculating the mass of the proton, or for that matter that of the electron, the energy of the electromagnetic field was taken for the rest mass of the particle. Such an assumption may not be strickly correct. A large fraction of the mass of the entities may arise from the magnetic interaction which the magnetic moment of the particle produces. A further objection arises from the fact that such a large value for b is inconsistent with the height of the proton barrier as computed from the observations of White [7] and Tuve Heydenberg and Hafstad.[8] The calculated height of the barrier agrees very well with a value for b, the limiting value of the electrostatic field of an isolated particle, which is the same as that of the electron. This conclusion indicates that the mass of the proton results from the spin properties of the particle rather than from its electrostatic field.

If the value of b for a proton does not differ appreciable from its value for an electron, then, for collisions in which the energy of the impinging proton is greater than 5×10^5 el.v., deviations from the scattering law calculated for a point entity should appear.

Our conclusion is that the facts concerning proton on

[5] W. Heisenberg and Euler, *Zeits. f. Phys.*, **98**, 714, 1936.
[6] M. Born, *Nature*, **136**, 952, 1935.
[7] M. S. White, *Phys. Rev.*, **49**, 309, 1936.
[8] M. A. Tuve, N.P. Heydenburg and L. R. Hafstad, *Phys. Rev.*, **49**, 402, 1936.

proton scattering are not inconsistent with the predictions of the unitarian field theory. The deviations of the potential field from that of the electrostatic field of an isolated particle arise first, from the finite value for b—this is equivalent to postulating a finite value for the hieght of the barrier—and secondly, in the region exterior to the barrier, from the influence of the magnetic field on the electric interaction. These conclusions are in harmony with the experimental evidence.

Finally, I wish to express my appreciation to Dr. W. F. G. Swann for his assistance in the preparation of this paper.

TEMPERATURE STRESSES IN FLAT RECTANGULAR PLATES AND IN THIN CYLINDRICAL TUBES.

BY

J. P. DEN HARTOG.[1]

INTRODUCTION.

A serious difficulty in the production of satisfactory structures by means of welding lies in the appearance of internal stresses which seem to be unavoidable in view of the large and localized temperature variations involved in the process. In many cases these stresses, which usually become as high as the yield point, are of no particular danger to the safety of the structure, since the superposition of additional service stresses causes the affected parts to yield without rupture, thus keeping the maximum stress no higher than the yield stress of the material. In fact, when properly managed, the residual stresses can even be made to serve a useful purpose, as is the case for instance in guns which are subjected to internal hydraulic pressure. This sets up internal stresses having a sign opposite to those of the subsequent service stresses, and these service stresses can now be made practically twice as high before a permanent deformation occurs. But in welded structures the residual stresses cannot be controlled so nicely, and once in a while a rupture will occur without previous yielding, with disastrous results. In order to understand these at present inexplicable failures, a closer study of the effect is desirable, and an extended program of research in this direction was started by the initiative of Professor C. A. Adams, Past-President and now an Honorary Member of the American Welding Society.

The program in its full scope comprises an experimental study of the temperatures involved as a function of location as well as of time, measurements of the residual stresses caused and of the effects of subsequent loads on the cold structure, a

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critical research into the efficacy of the various methods of annealing, etc. Analytical investigations paralleling these tests are also contemplated with the object of obtaining a better insight into the phenomena.

As a preliminary to this ambitious undertaking, it is necessary to know the temperature stresses which are set up in a number of simple structures, because a calculation of residual stress necessarily starts from the temperature stresses set up in the unevenly heated object. In this paper the solutions are given for the following cases:

I. A rectangular plate heated along a center line (representing the case of two equal plates welded together along one side).
II. A large flat plate with a central circular hot spot (referring to spot welding).
III. A tube with a hot circle in the center and cool at its ends (of interest for the case when two tubes are butt-joined by welding).
IV. A tube with a hot generating line (referring to the case of a tube being made of a rolled-up plate by welding along a seam).

The general method for the solution of the problems (II) and (III) is well known, the object of this paper being only to carry the calculations to a numerical conclusion for a number of interesting cases and to present the results in a form useful for future reference (see Figs. 12 and 14). For the first problem a solution is obtained which is believed to be new and of some interest for two-dimensional elasticity in general. It is given in detail in Appendix I, and the results appear in the Figs. 4, 5 and 6. The fourth problem mentioned above is shown to be virtually the same as the first problem.

In all results the stress has been expressed in terms of $\alpha E T_0$, where

α = coefficient of linear thermal expansion,
E = modulus of elasticity,
T_0 = temperature difference, usually between the highest and average temperatures in the structure.

This quantity $\alpha E T_0$ has a simple physical meaning. It is the

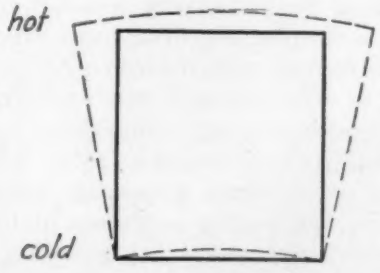
stress set up in a simple bar, when it is heated uniformly to the temperature T_0 and prevented from expanding longitudinally by compressive forces at the ends. In a structure which is heated non-uniformly the various elements tend to expand with the temperature but are partially prevented from doing so by the neighboring elements. This sets up the " temperature stresses " and it is therefore rational to express these stresses in terms of the quantity $\alpha E T_0$.

For a bar in uniform tension the maximum shear stress is half as great as the tensile stress, so that a tension or compression of $\alpha E T_0$ corresponds to a shear stress of $\alpha E T_0/2$.

I. RECTANGULAR PLATE.

There is a theorem (see Timoshenko's " Theory of Elasticity," p. 204) stating that in a body with a linear temperature distribution no heat stresses can exist. In particular, if we consider a rectangular plate hot at the top edge and cold at the bottom, with a linear temperature diagram in between (Fig. 1), we recognize that the top fibres will tend to

FIG. 1.

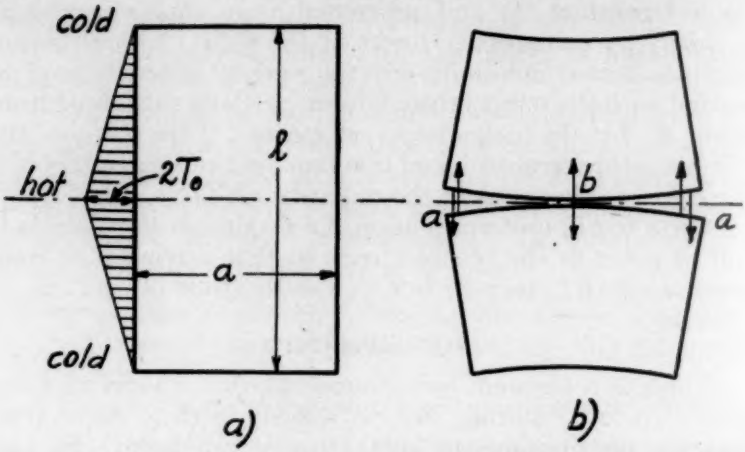


A rectangular plate with a linear temperature distribution bends freely without temperature stress.

expand. This can take place unobstructed if the plate bends as indicated by the dotted lines in Fig. 1. In that case no stresses will be set up.

Next consider the plate of Fig. 2*a*, which is hot along the center line and cold at its ends. If that plate were cut along its hot line, the two halves would bend as shown in Fig. 2*b*, and no stresses would be set up anywhere. In joining the two halves together again, it is seen that we need tension

FIG. 2.



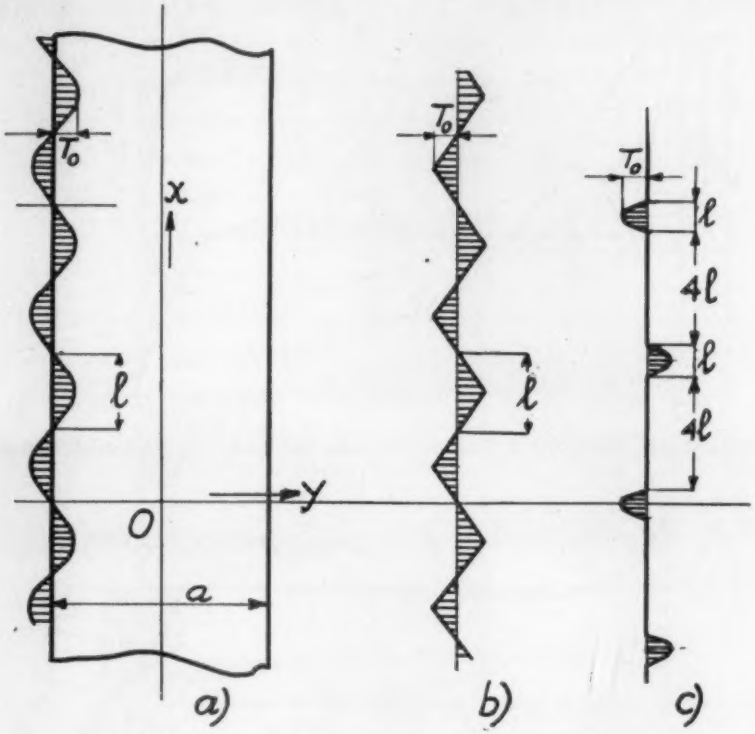
Temperature stress in a plate with a hot center line.

stresses at the edges a,a and compression stresses near the center b. These stresses in the uncut plate represent the temperature stresses.

The problem of Fig. 2 is very difficult to solve and we therefore tackle a simpler one first, from which we can later draw certain conclusions with regard to Fig. 2. This simpler problem is that of a flat plate of small thickness, of width a and of *infinite* length, having a sinusoidal temperature distribution $T = T_0 \sin(\pi x/l)$ along its sides, whereas the temperature is constant on lines across the width of the plate. This temperature distribution is shown in Fig. 3*a*, and the maximum temperature stress $\sigma_{max.}$ occurring in the plate is represented by curve A in Fig. 4 as a function of l/a, which is the ratio of a half wave-length of the temperature variation to the plate width. It is seen that this stress is the full amount $\alpha E T_0$ in case the half wave-length l of the temperature variation is rather short with respect to the width a of the plate. For longer wave-lengths the maximum temperature stress is seen to diminish.

In case the temperature distribution along the strip of infinite length is not sinusoidal, but expressed by a zig-zag curve with sharp points (Fig. 3*b*), the maximum temperature

FIG. 3.



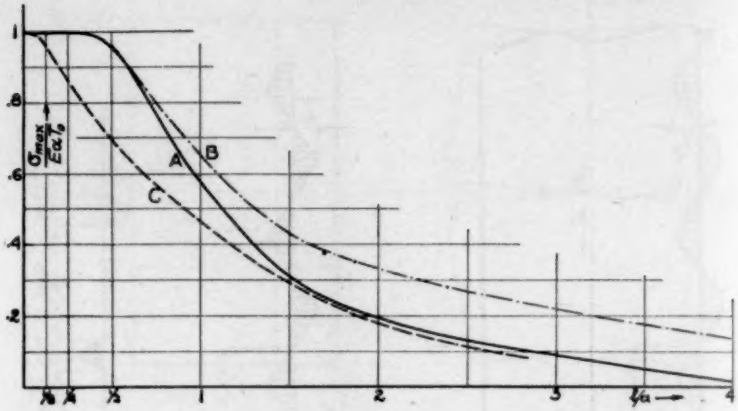
Three temperature distributions along an infinitely long strip.

stress at longer wave-lengths is considerably greater, as shown by the dash-dot curve *B* of Fig. 4. *For short wave-lengths, however, the maximum stress is completely independent of the type of temperature curve; it is determined solely by the maximum temperature difference, and not by the temperature gradient.*

A third distribution along the infinite strip which has been calculated consists of isolated hot and cold spots of sinusoidal shape at rather large distances apart (Fig. 3*c*). The dashed curve *C* of Fig. 4 gives the maximum temperature stress for this case.

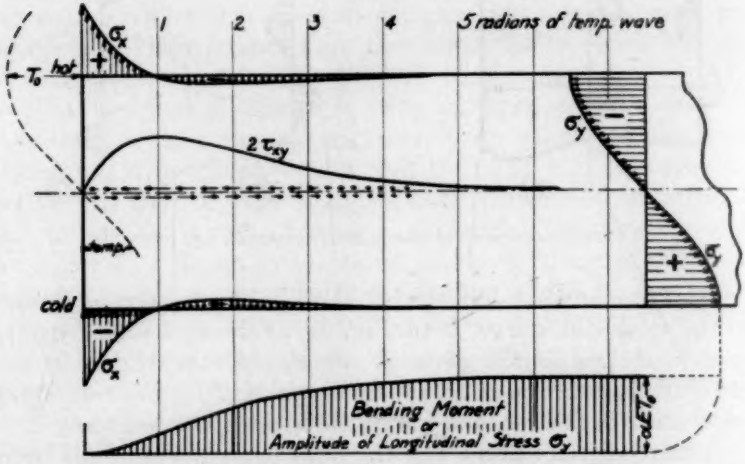
While Fig. 4 shows the greatest stress that occurs in the plate, the Figs. 5 and 6 serve to give a physical understanding of the stress distribution for a sinusoidal temperature along

FIG. 4.



Maximum temperature stress in an infinite strip for the three temperature distributions of Fig. 3.

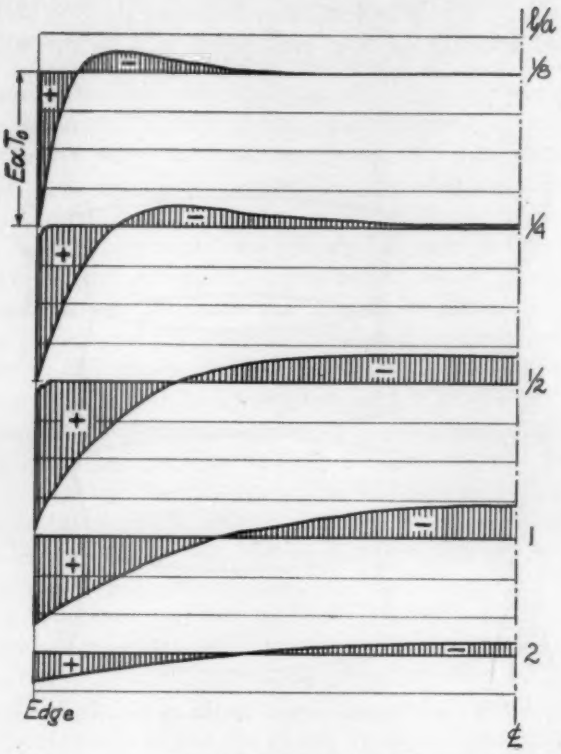
FIG. 5.



Stresses in the strip of Fig. 3a for very short wave-lengths ($2 \ll a$).

the infinite plate (Fig. 3*a*). Imagine that plate to be cut up in a series of strips along the cross-lines of maximum and of minimum temperature, the lengths of the individual strips being the original width a of the plate and the breadth of the strips being half the wave-length of the temperature distri-

FIG. 6.



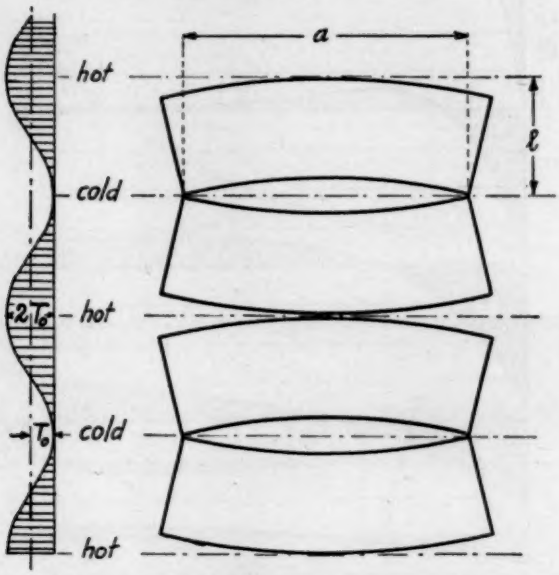
Stresses across the hot line of the strip Fig. 3a for various wave-lengths of the temperature distribution.

bution. In each of these strips the temperature distribution does not deviate much from a linear one and the strips would bend as indicated in Fig. 7. In order to fit these various strips together again, forces have to be exerted on the cut surfaces. From Fig. 7 it is clear that near the edges of the plate we expect tensile stresses at the hot spots and compressive stresses at the cold spots, whereas near the center line of the plate we expect the opposite.

In case the wave-length l is short in comparison to the width a of the plate, the strips of Fig. 7 become very thin and can be considered as long "beams." When cut loose from the rest of the plate, each of these beams will bend practically

in a circular arc because the hot side will elongate more than the cold side. The problem then becomes that of finding the distributed loading on the long sides of a beam which will

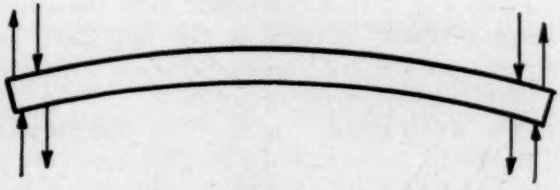
FIG. 7.



The infinite strip of width a cut along its lines of maximum and minimum temperature.

straighten out the originally uniform curvature (Fig. 8), or, which is the same, that of finding the loading which will bend an originally straight beam into a circular arc. From the strength of materials we know that a beam can be bent circularly by a bending moment which is *constant* along its

FIG. 8.



Localized bending moments at the ends of a beam giving a constant bending moment in the center portion.

length, and since any " loading " on the beam is associated with a variation in bending moment along the length, the problem is unsoluble as stated. In the center portion of the beam of Fig. 8 (which is one of those of Fig. 7), we can obtain pure circular bending by applying to the ends fairly concentrated tension and compression loadings close to each other, which together form a bending moment. The exact distribution of these forces is disclosed by the analysis of Appendix I, and is shown in Fig. 5.

This figure represents the left end of one of the beams of Fig. 7, where the half wave-length is very short in comparison to the width of the plate, so that the beam is very long. The top fibre is hot and the bottom fibre cold. The σ_x-stress distribution shown represents the localized bending moment at the end, which is seen to have died out at approximately half a wave-length from the end of the beam. At greater distances from the end there are no tensile stresses on the beam (i.e. no tensile stresses in the longitudinal direction of the long plate made up of all the thin beams). Also there is a bending moment in the beam at some distance from the ends, as shown by the σ_y-diagram. Since near the center of the beam its fibres are completely prevented from expanding in the y-direction and are free to expand in the x-direction, it is clear that the maximum bending stress σ_y there equals the full value $\alpha T_0 \cdot E$. The bending moment of the σ_y-stresses is obviously zero at the left free end of the beam, because there cannot exist any σ_y-stresses there. The analysis shows that the σ_y-distribution across a vertical in the diagram (Fig. 5) is always sinusoidal with an amplitude as shown at the bottom of the figure.

In the center line of the beam (i.e. along a line of neutral temperature), there are some shear stresses τ_{xy}, which again die out at some distance from the end. The maximum value of that shear stress is 0.37 $\alpha T_0 E$, corresponding to a tensile stress of 0.74 $\alpha T_0 E$ as plotted. The greatest stress, therefore, is the σ_x at the end or the σ_y in the center portion of the beam.

When the half wave-length l of the temperature distribution increases with respect to the width a, the beams of Fig. 7 become higher and comparatively shorter, so that the ordinary " beam " notions can be applied no longer. In particular, the

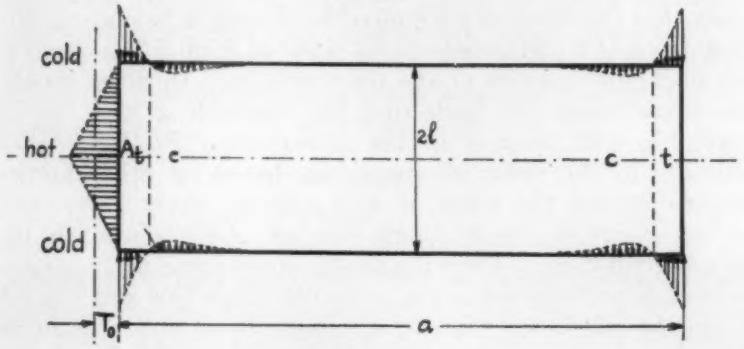
concentrated normal stresses near the ends as shown in Figs. 8 and 5 do not occur any more in that form. Figure 6 gives the normal (σ_x) stress distribution across a hot line in the plate for various wave-lengths. The ratio l/a is half the wave-length of the temperature distribution divided by the plate width, or the beam height over beam length of Fig. 7. For a ratio 1/8 the distribution of Fig. 5 (which is for $l/a = 1/\infty$) still applies, but for shorter beams there is more and more deviation. For a beam twice as high as it is long ($l/a = 2$), the distribution is completely different. In the analysis of Appendix I, it is found that the highest stress occurring in the structure is the σ_x in the left extremity of the diagrams of Fig. 6, rather than the σ_y near the center of the beam. It is this σ_x stress which has been plotted as a function of l/a in Fig. 4. The peak stress is independent of l/a for wave-lengths which are so short that the simple " beam " theory applies. Outside that region the stress falls off with increasing wave-length.

The solution obtained for sinusoidal temperature distribution along an infinite strip, as shown primarily in Figs. 5 and 6, has been made the starting point for finding the solution to other problems. For instance, the infinite strip with the broken linear temperature distribution of Fig. 3*b* has been solved by using a Fourier series. The zigzag line is considered as made up of a series of sine waves of various wave-lengths, for each of which the stress distribution is known from Figs. 5, 6 or from Eq. (3). The superposition of all these stresses gives the stress for the zigzag temperature. It was thus found that for short zigzag wave-lengths the σ_x-diagram of Fig. 5 applies practically without change. The peak stress remains exactly $\alpha E T_0$, the zero point of the diagram shifts from $y = 1$ in Fig. 5 to $y = 0.96$, and the negative maximum of $0.135\ E\alpha T_0$ (at $y = 2$ approximately) increases by less than one per cent. of its value. Thus the error committed is insignificant if we assume the σ_x-diagram of Fig. 5 to apply to the zig-zag temperature distribution as well.

Now we are ready to return to the problem we set out to solve initially, namely that of the plate of *finite* length, Fig. 2. This plate is made up of two of the " beams " of Fig. 7 or 5. If cut out of the infinite plate, there are compressive and

tensile σ_z forces on the upper and lower boundary as indicated in Fig. 9. However, in the finite plate these stresses do not exist and we can spirit them away by superposing on Fig. 9 the opposite set of forces, i.e. tension at the very ends of the

FIG. 9.

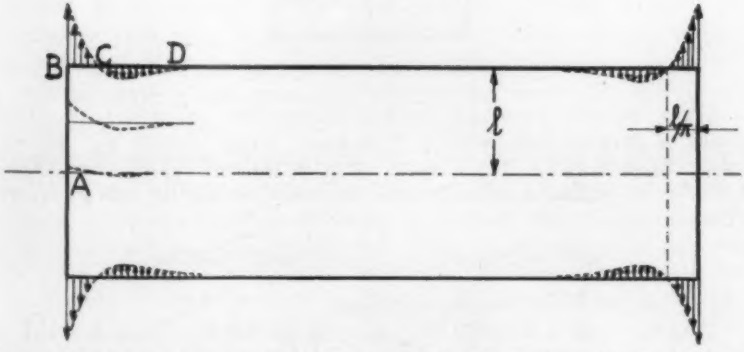


Rectangle of one wave-length cut out of infinite strip with zig-zag temperature. The stresses shown are those occurring in the infinite strip.

beam and compression nearer the middle. *The actual tensile stress at the dangerous point A in the finite plate then will consist of the stress indicated in curve B of Fig. 4 and of an additional stress caused by the tension-compression superposed on the free edges* (Fig. 10).

The problem of the determination of this additional tensile

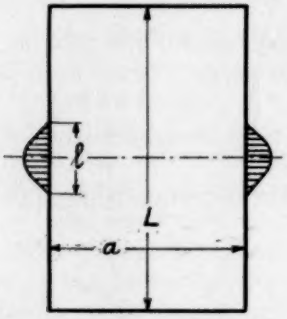
FIG. 10.



Stresses which have to be superposed on Fig. 9 in order to make the top and bottom stress free.

stress at A is one which cannot be solved exactly. However, we notice first that the tension-compression system BCD on the top of the plate is balanced in itself and consequently has no resultant. Moreover, AB is 3.14 times the distance BC and at the point D, where $BD = BA$, the compressive stress has practically died out. By Saint Venant's principle, we know that the effect of a balanced force system has practically died down at a distance from its point of application equal to the linear dimensions of the force system. In other words, the stress along the horizontal line through A (Fig. 10) is negligible with respect to the stress along BCD. A good estimate of the order of magnitude based on other known results [2] is that the stress at A is approximately 4 per cent. of the B-stress. Since along the hot A-horizontal in the *infinite* strip (Fig. 9) there is a tensile stress equal and opposite to the compressive stress along the cold lines (see also Fig. 5), it may be concluded that *the maximum stress occurring in the*

FIG. 11.



Rectangular plate with heated center strip and large cold end portions.

[2] For a sinusoidal stress along the strip (instead of the one of Fig. 9 or 10) the solution is given in Timoshenko's "Elasticity," p. 46, formula (k). In that formula, by setting $A = B$, $y = 0$, and $\alpha c = 2\pi$, the case becomes comparable to our Fig. 10, and the σ_y-stress becomes (after considering that $\sinh 2\pi \approx \cosh 2\pi \gg 2\pi$):

$$\sigma_y = -A \sin \alpha x \cdot \frac{2\pi + 1}{\sinh 2\pi} = -0.027\, A \sin \alpha x,$$

being 2.7 per cent. of the maximum σ_x-stress.

A similar result is obtained by considering concentrated loads as shown in Fig. 36, page 50, of the source quoted.

finite plate of Fig. 2a can be found by increasing by some 4 per cent. the ordinates of curve B, Fig. 4.

Another case of practical interest is that of a rectangular plate which is hot only in its center part l and of which considerable end portions remain cold, as shown in Fig. 11. If the cold end portions are sufficiently large ($L \gg l$) the stress distribution will be the same as that of an infinite strip with periodic hot and cold spots at distances L part (Fig. 3*c*). The solution for such a case again can be found from that of the sinusoidal case by a Fourier series analysis, and the result is given in curve C of Fig. 4 (see Appendix I, *c*).

II. CIRCULAR HOT SPOT IN PLATE.[3]

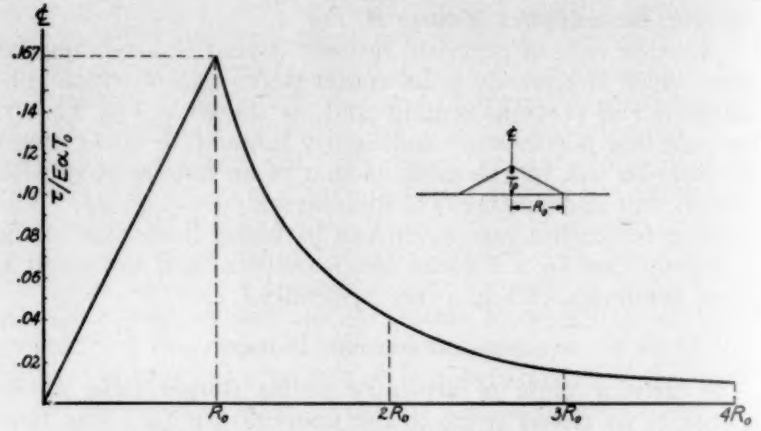
A circular plate of arbitrary radius (larger than R_0) is heated at its center point to the temperature T_0. The temperature then falls off linearly with the radial distance, is zero at the radius R_0, and remains zero outside that circle in the rest of the plate. The temperature, if plotted perpendicularly to the plate, would thus be represented by a cone of height T_0 and having a base circle R_0.

The inside hot material will try to expand and is partially prevented from doing so by the cold outside part of the plate. Thus there will be set up a radial compressive stress which is zero at the outside and maximum at the center. Also the various points of the disc will move outwards due to the hot spot in the center so that each concentric circle will expand somewhat. Wherever this expansion is greater than the heat expansion (as in the cold outside material), there will be a tensile hoop stress. The combination of the radial compression and the hoop or tangential tension causes a shear stress, which is shown in the diagram of Fig. 12.

The remarkable property of this diagram is that *it applies to all sizes of plate.* In other words, for a plate of radius R_0 heated all over with a conical temperature distribution, only the central linear portion of the curve expresses the shear stress, and for a plate of infinite extent the diagram in its

[3] The complete analysis for this case is given in Timoshenko's "Elasticity," pp. 364–366, to which the reader is referred. It seems that the final formulas (232) to (234) contain a slight inaccuracy. The factor $(1-\nu)/(1-2\nu)$ which they all contain should be dropped.

FIG. 12.



Shear stress in a circular plate with a central hot spot. The validity of the diagram is independent of the size of the plate or of the extent of the hot spot.

entirety (extrapolated to ∞) holds. For a plate between these two extreme cases, that part of the diagram up to the outside radius applies.

For a circular plate with a "conical" temperature distribution the maximum shear stress occurs at the edge of the temperature rise and is $\alpha T_0 E/6$, independent of the size of the plate or of the size of the heated circle. In particular, this means that the stress is independent of the temperature gradient, but is determined by the maximum temperature difference alone.

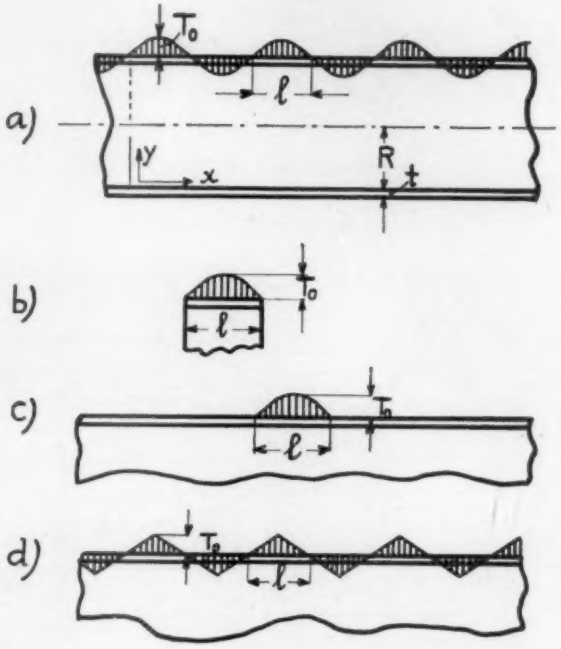
The fact that the shear stress diagram is independent of the size of the plate should not be construed as meaning that the whole stress distribution is independent of that quantity. The maximum shear stress at the periphery of a plate of radius R_0 is made up completely of hoop or tangential stress with the radial stress absent, whereas for a plate of infinite extent the stress at R_0 is half hoop stress (tensile) and half radial stress (compressive). In both cases, however, the resultant shear stress is the same.

III. CYLINDRICAL TUBE WITH HOT CIRCLE.

A problem very similar to the rectangular plate is that of a tube which is heated along a circle lying in a plane perpendicular to its axis. Again it has been found analytically con-

venient to begin with a solution of the problem of an infinite tube with a sinusoidal temperature distribution all along its length, as shown in Fig. 13, *a*. The maximum stress thus set up is located at the spot of highest temperature, and is composed partly of hoop compression (because the hot circle

FIG. 13.

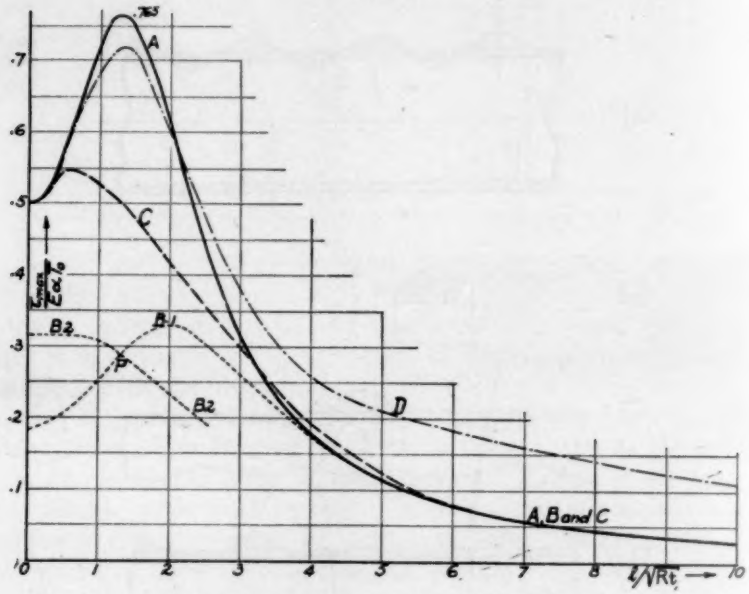


Four temperature distributions along a thin cylindrical tube.

cannot expand radially as much as it wants to) and partly of bending along a generator of the tube. For very long wave-lengths of the temperature, obviously both these stresses must become small. The half wave-length l of the temperature variation in this connection is best expressed in terms of the combination $\sqrt{Rt}$, where R is the tube radius and t its thickness. For "zero" (i.e. very short) wave-lengths, the tube will not deform at all and the stress consists of a hoop compression of the full amount $\alpha E T_0$ at the hot spots and of a hoop tension $\alpha E T_0$ at the cold spots, because the material is

prevented completely from expanding or shrinking. For wave-lengths slightly longer than "zero" the tensile (or compressive) stress is practically $\alpha E T_0$ yet, but an additional bending comes in, which makes the total stress greater than for $l = 0$. All this is shown in the full curve A of Fig. 14.

FIG. 14.



Maximum temperature stress for the tubes of Fig. 13.

For a *finite* tube of length l, hot in the center and cold at the ends with a half sine-wave of temperature (Fig. 13*b*), the result is expressed by the dotted curve B, consisting of two parts, 1 and 2. For long wave-lengths the maximum stress occurs at the center of the tube as shown by the 1-branch, while for short wave-lengths the stress at the free ends is greater, which is represented by the 2-branch. For very short wave-lengths, the generators of the cylinder cannot bend and the whole tube will expand to a diameter corresponding to the average temperature $(2/\pi)T_0$. This causes compression in the center of the tube (the expansion being insufficient for the high temperature there) and tension at the cold ends. It

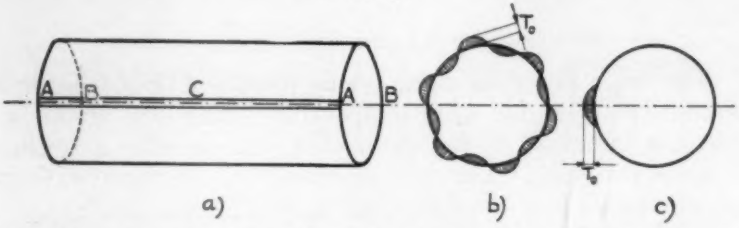
can thus be understood that the stress at the ends is $\sigma = (2/\pi) \cdot \alpha E T_0$ and the shear stress is $\tau = (1/\pi) \cdot \alpha E T_0$ as shown by the left-end point of curve *B2* in Fig. 14.

For an infinitely long tube with a single hot half sine-wave of length l, the stress is expressed by curve *C*, while finally curve *D* in Fig. 14 gives the stress in an infinite tube for a continuous zigzag or broken-linear temperature distribution, as shown in Figs. 13*c* and *d*.

IV. TUBE WITH HOT GENERATING LINE.

If a longitudinal seam is welded in a tube, the resulting temperature distribution is roughly constant along the length, but varies with the angular position on the circumference of a circle. Figure 15*c* shows this condition, but a number of other

FIG. 15.



Tube with temperature constant along the length, but depending on the angular position (hot seam).

distributions, as the sinusoidal one of Fig. 15*b*, the zigzag one, etc., also will be discussed.

Let us cut the tube along *BB*, the generator opposite to the heated seam, and let us flatten out the tube into a plate, which then is completely identical to that of Fig. 2. Now we compare the possibility of strain in the three directions (longitudinal, tangential and radial) in the two cases. First it is recognized that the oppositions to longitudinal strain in the plate and in the cylinder are completely identical. In the case of a long tube, the condition at the center *C* in the flat plate is one of longitudinal stress only, as can be seen from Fig. 5, because there is no opposition whatever to lateral (tangential) expansion of the plate. For the plate rolled up into a tube, there *is* some constraint. Imagine in Fig. 15 the cold tube completely without stress. The heating of the

seam causes it to expand tangentially, which is opposed by the bending stiffness of the ring. The problem is the same as that of driving a longitudinal wedge into the seam cut open. For a flat plate, this is done without resistance: the two half plates merely move apart; for the ring there is some resistance. In Appendix II*e* it is shown, however, that this resistance is completely negligible.

The third or radial strain, in the case of a flat plate, becomes perpendicular to the thickness and is absent. In the tube, however, such a strain does occur and causes bending stresses. But again it is shown in the Appendix that these stresses, though not quite as small as the tangential compressive stresses, are also negligible (less than one per cent. of $\alpha E T_0$ in most cases).

Thus it may be concluded that *the results shown in the Figs. 4, 5 and 6 are applicable to the seamed tube as well as to the flat plate.*

COMPARISON AND CONCLUSIONS.

The main practical value of the results obtained lies in a possibility of judging what temperature differences are allowable in a structure during the cooling process after annealing or stress relieving. For mild steel not too far removed from room temperature, the following numerical data apply:

$$\begin{aligned} \text{modulus } E &= 30 \cdot 10^6 \text{ lbs./in.}^2 \\ \text{expansion coefficient } \alpha &= 6.7 \cdot 10^{-6} \text{ in./in. deg. F.} \\ \text{yield point } \sigma_{\text{yield}} &= 30{,}000 \text{ lbs./in.}^2; \\ \tau_{\text{yield}} &= 15{,}000 \text{ lbs./in.}^2 \end{aligned}$$

The modulus and the yield point both go down with increasing temperatures, the latter, however, more sharply than the former. With these figures we arrive at the result that:

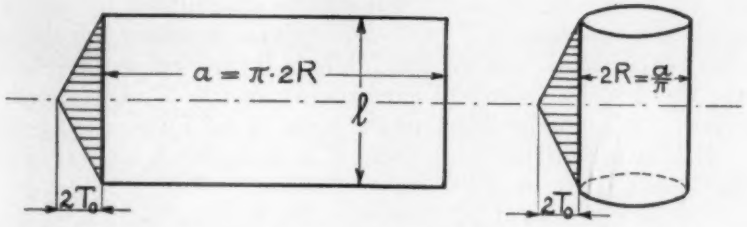
$$\alpha E = 200 \text{ lbs./in.}^2 \text{ deg. F.}$$

or, in words, a bar heated to one degree F., which is prevented from expanding longitudinally, will experience a compression of 200 lbs./in.^{2} Thus *if complete prevention of expansion in one direction takes place* ($\sigma/E\alpha T_0 = 1$ or $\tau/E\alpha T_0 = 0.5$), *a temperature difference of 150 deg. F. is required to start yielding* (near room temperature).

In this light, the temperature stress in a hot spot of a round plate, being one-third of this maximum value, does not seem very serious.

The results for the rectangular flat plate or for the tube with hot generator on one hand and those for the tube with hot circle on the other hand, as given in the Figs. 4 and 14, appear quite similar to one another: the stress is the full one $E\alpha T_0$ (or somewhat larger) at short wave-lengths and falls off considerably at longer wave-lengths. However, there is a decided difference between the two, residing in the fact that practical plate cases have their abscissas in the left side of the diagram with high stresses, whereas the corresponding tube cases have much larger relative wave-lengths, and lower stresses. As an example, consider a plate of $l/a = 1/2$, as shown in Fig. 16. By Fig. 4 this plate will have practically

FIG. 16.



Comparison between a flat plate and that same plate rolled up into a tube.

the full temperature stress $\alpha E T_0$. Now let this plate be rolled up into a tube, and let the thickness of the material be 1/25th of the radius of the tube. Then, for the tube,

$$l/\sqrt{Rt} = l/\sqrt{R^2/25} = 5l/R = 5l/a/2\pi = 10\pi\, l/a = 5\pi = 15.4,$$

which is represented by a point way off the right side of Fig. 14, where the stress is entirely negligible.

Thus we see that *the stresses in a flat plate or in a hot-generator-tube are very much more serious than those in a hot-circle-tube of comparable dimensions having a comparable temperature distribution.*

APPENDIX I.

Flat Plates.

(*a*) *Sinusoidal Temperature Distribution along an Infinite Strip.*—Lay the origin of coördinates in the center line of the strip (Fig. 3*a*) and consider a sinusoidal temperature distribution $T = T_0 \sin \omega x$, independent of y. By the general theorem on heat stresses, as stated for instance in Timoshenko's "Theory of Elasticity," p. 206, the stress consists of the sum of three parts:

(1) a two-dimensional hydrostatic compression of $\{\alpha E T_0/(1-\nu)\} \sin \omega x$ at each point,
(2) body forces $X = -\{\alpha E T_0/(1-\nu)\}\omega \cos \omega x$,
(3) surface tensions $\{\alpha E T_0/(1-\nu)\} \sin \omega x$.

It will be noted that we write here $(1-\nu)$, whereas in the source quoted the formulas have $(1-2\nu)$. This is because the problem considered by Timoshenko is three-dimensional, and the temperature expansion can be annihilated by an all-sided compression $\alpha E T/(1-2\nu)$. Here, however, we have a flat plate, which can expand freely in its thickness direction, and the expansion due to a temperature rise T can be neutralized by a compression *in the plane* of $\alpha E T/(1-\nu)$.

It is seen that the body force X is derivable from a potential $V = \{\alpha E/(1-\nu)\} \cdot T$, or

$$X = -\frac{\partial V}{\partial x}, \qquad Y = 0 = -\frac{\partial V}{\partial y}.$$

This makes it possible to solve the stress problem by means of a stress-function Φ (Timoshenko's "Elasticity," pp. 25–26), satisfying the differential equation:

$$\nabla^4 \Phi = -(1-\nu)\nabla^2 V \tag{1}$$

or:

$$\frac{\partial^4 \Phi}{dx^4} + 2\frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = \alpha E T_0 \omega^2 \sin \omega x.$$

The solution of this equation is any biharmonic function plus a particular integral, which we assume in the form $A \sin \omega x$. Substituting this and solving for A, the particular integral becomes:

$$\Phi_{\text{particular}} = \frac{\alpha E T_0}{\omega^2} \cdot \sin \omega x.$$

Assume now the biharmonic function to have the form $f(y)\cdot\sin \omega x$ and substitute. The $\sin \omega x$ can be divided out and the partial differential equation reduces to an ordinary one:

$$f^{(4)} - 2\omega^2 f'' + \omega^4 f = 0.$$

The solution of this, containing four integration constants, is known. Together with the particular solution, we thus find for Φ:

$$\Phi = \sin \omega x\left[(C_1 y + C_2)e^{\omega y} + (C_3 y + C_4)e^{-\omega y} - \frac{\alpha E T_0}{\omega^2}\right] \cdots (2)$$

from which the stresses can be derived by

$$\sigma_x = V + \frac{\partial^2 \Phi}{\partial x^2}, \qquad \sigma_y = V + \frac{\partial^2 \Phi}{\partial y^2}, \qquad \tau_{xy} = -\frac{\partial^2 \Phi}{\partial x \partial y}.$$

These are the stresses due to the body forces X and the surface tensions only. The general two-dimensional compression $\alpha E T/(1 - \nu)$ has to be superposed. For the temperature stresses so found, we have the following boundary conditions:

$$\tau_{xy} = \sigma_y = 0, \quad \text{at} \quad y = \pm a/2$$

or, we may take the conditions at $y = +(a/2)$ only, if we see to it that the stress distribution is made symmetrical about the x-axis. This can be accomplished by making $C_2 = C_4$ and $C_1 = -C_3$ in the stress function. With that the (symmetrical) temperature stresses σ_y and τ_{xy} become:

$$\sigma_y = \sin \omega x\left[\frac{\nu}{1 - \nu}\alpha E T_0 - 2C_1\omega^2 y \sinh \omega y - 2C_2\omega^2 \cosh \omega y\right],$$

$$\tau_{xy} = -\omega\cdot\cos \omega x[2C_1\omega y \cosh \omega y + 2(C_1 + C_2\omega)\sinh \omega y].$$

Substituting $y = a$ and equating these expressions to zero gives two linear equations from which C_1 and C_2 can be solved:

$$C_1 = +\frac{\alpha E T_0}{2\omega}\frac{\sinh(\omega a/2)}{\omega a/2 + \sinh(\omega a/2)\cdot\cosh(\omega a/2)},$$

$$C_2 = -\frac{\alpha E T_0}{2\omega^2}\frac{\sinh(\omega a/2) + (\omega a/2)\cdot\cosh(\omega a/2)}{\omega a/2 + \sinh(\omega a/2)\cdot\cosh(\omega a/2)}.$$

This gives us the final expressions for the temperature stresses as functions of the location x,y and of the parameter $\omega a (= \pi a/l$, where l is half the wave-length of the temperature wave):

$$\left.\begin{aligned}
\sigma_x &= \alpha E T_0 \sin \omega x \\
&\quad \times \sinh(\omega a/2)\cdot \omega y \sinh \omega y + \\
&\quad + \{\sinh(\omega a/2) - (\omega a/2)\cosh(\omega a/2)\}\cdot \cosh \omega y \\
&\quad \cdot \frac{}{\omega a/2 + \sinh(\omega a/2)\cdot\cosh(\omega a/2).} \\
\sigma_y &= \alpha E T_0 \sin \omega x \Big[-1 + \\
&\quad - \sinh(\omega a/2)\cdot \omega y \sinh \omega y \\
&\quad + \frac{+ \{\sinh(\omega a/2) + (\omega a/2)\cosh(\omega a/2)\}\cosh \omega y}{\omega a/2 + \sinh(\omega a/2)\cdot\cosh(\omega a/2)}\Big] \\
\tau_{xy} &= -\alpha E T_0 \cos \omega x \\
&\quad \times \sinh(\omega a/2)\cdot \omega y \cosh \omega y \\
&\quad \cdot \frac{- \omega a/2 \cdot \cosh(\omega a/2)\cdot \sinh \omega y}{\omega a/2 + \sinh(\omega a/2)\cdot\cosh(\omega a/2)}
\end{aligned}\right\} . \quad (3)$$

These expressions find their graphical interpretation in the Figs. 4, 5 and 6. It is noticed that they are all sinusoids with respect to the x or longitudinal direction, but are more complicated along the y-direction. It can also be checked that σ_x is the y-derivative of τ_{xy}, and again that τ_{xy} is the y-derivative of σ_y. For very short wave-lengths l the "beam"-interpretation of Fig. 5 or 8 holds, and then the three stresses σ_y, τ_{xy} and σ_x behave as the bending moment, shear force and normal load respectively, which have to be derivatives of each other. This relation is clearly shown in Fig. 5.

(*b*) *Zig-Zag Temperature Distribution in an Infinite Strip.*— When the temperature along the strip is no longer sinusoidal, but still periodic, the problem can be solved by expanding the temperature wave in a Fourier series, calculating the stress of each harmonic component by the formulas (3) and summing up the various stresses.

The first case to be considered is that of a zig-zag wave (Fig. 3*b*), of half wave-length l. The fundamental frequency is found from the relation: $\omega l = \pi$. The Fourier series for

such a curve is:

$$T = \frac{8}{\pi^2} T_0 \left[\cos \omega x + \frac{1}{9} \cos 3\omega x + \frac{1}{25} \cos 5\omega x + \cdots \right]$$

or:

$$T = \frac{8T_0}{\pi^2} \sum_{1,3,5,\cdots} \frac{1}{n^2} \cos n\omega x. \tag{4}$$

The maximum stress in a plate with a sinusoidal temperature distribution is the σ_x at the point $\omega x = (\pi/2)$ and $y = (a/2)$ (see Fig. 6), which from the stress-formula (3*a*) is:

$$\sigma_{x\,\max.} = \alpha E T_0 \cdot \frac{-\omega a/2 + \sinh(\omega a/2) \cosh(\omega a/2)}{+\omega a/2 + \sinh(\omega a/2) \cosh(\omega a/2)} = \alpha E T_0 \cdot \frac{-\omega a + \sinh \omega a}{+\omega a + \sinh \omega a}. \tag{5}$$

From this formula the curve *A* of Fig. 4 was plotted. The maximum stress for a zig-zag temperature distribution (curve *B* of Fig. 4) is found from Eq. (5) by summing the Fourier series (4) graphically. For instance, consider the point $l/a = 3$ in Fig. 4. A zig-zag wave of amplitude T_0 consists first of a fundamental wave of amplitude $(8/\pi^2)T_0$, then of a third harmonic 1/9th as large, etc. The fundamental stress by curve *A*, Fig. 4, is: $(8/\pi^2) \times 0.09$. The third harmonic has a wave-length three times as short or $l/a = 1$, with a stress of 0.57 per unit of temperature amplitude. For our third harmonic this becomes $(8/\pi^2) \times (1/3^2) \times 0.57$, for the fifth $(8/\pi^2) \times (1/5^2) \times 0.88$, and for the seventh $(8/\pi^2) \times (1/7^2) \times 0.98$. The still higher harmonics lie on the horizontal part of the stress curve. Summing up all these amounts leads to a stress of:

$$\frac{8}{\pi^2}\left[0.09 + \frac{0.57}{9} + \frac{0.88}{25} + \frac{0.98}{49} + \frac{1}{81} + \frac{1}{121} + \cdots\right] = \frac{8}{\pi^2} \times 0.270 = 0.22$$

which is the ordinate of the point $l/a = 3$ of curve *B*. In this manner all the other points were calculated.

The σ_x-curve of Fig. 5 is of some interest as it represents

the manner in which a pure bending moment is given to a beam by normal stresses on its long sides. We want to see how this curve is affected by replacing the sinusoidal temperature by a broken linear one. The general equation of σ_x is (3*a*), but for short wave-lengths ($\omega a \gg 1$) this expression can be simplified. First introduce as a new variable $z = (a/2) - y$, i.e., take the origin at the edge of the plate instead of in the center line. Then, since $\omega a \gg 1$, we have

$$\sinh(\omega a/2) \approx \cosh(\omega a/2) \approx \tfrac{1}{2}e^{\omega a/2} \gg \omega a/2.$$

With this, the expression for σ_x at $\omega x = \pi/2$ simplifies to:

$$\frac{\sigma_x}{\alpha E T_0} = (1 - \omega z)e^{-\omega z} \tag{6}$$

which is the equation of the curve in Fig. 5 and of the top curve in Fig. 6. The *n*th harmonic of the temperature distribution will give a stress:

$$\sigma_x = \alpha E T_{n\text{th harm.}}(1 - n\omega z)e^{-n\omega z}$$

and for the diagram of Fig. 3*b*, the total stress is:

$$\sigma_x = \alpha E T_0 \frac{8}{\pi^2}\left[(1 - \omega z)e^{-\omega z} + \frac{1}{3^2}(1 - 3\omega z)e^{-3\omega z} + \frac{1}{5^2}(1 - 5\omega z)e^{-5\omega z} + \cdots\right]$$

or:

$$\sigma_x = \alpha E T_0 \frac{8}{\pi^2}\sum_{1,3,5}\frac{1}{n^2}(1 - n\omega z)e^{-n\omega z}. \tag{7}$$

The first harmonic is the top curve of Fig. 5. The third harmonic has 1/9th its amplitude and is three times as short horizontally. The 5th harmonic has 4 per cent. of the amplitude and is compressed horizontally to 1/5th of the width. This makes it clear that the convergence of the series (7) is very rapid and the final σ_x-curve for a broken-linear temperature distribution differs very little from the one shown in Figs. 4 and 5. The peak $E\alpha T_0$ is not changed at all; the zero point moves to the left from 1 to 0.96, and the compression maximum of $0.135 E\alpha T_0$ increases by less than one per cent of its value.

(*c*) *Single Hot Spot on Long Plate.*—Next consider a temperature distribution

$$T = T_0 \cos \frac{\pi x}{l} \text{ for } -l/2 < x < l/2, \quad \text{for } \frac{9l}{2} < x < \frac{11l}{2}, \quad \text{etc.}$$

and $T = 0$ for other values of x, as shown in Fig. 3*c*. When this function is developed in a Fourier series by the usual procedure, the result is:

$$\left.\begin{aligned} \frac{T}{T_0} = {} & 0.254 \cos \omega x + 0.234 \cos 3\omega x + 0.200 \cos 5\omega x \\ & + 0.155 \cos 7\omega x + 0.109 \cos 9\omega x + 0.063 \cos 11\omega x \\ & + 0.026 \cos 13\omega x + 0 - 0.014 \cos 17\omega x \\ & - 0.018 \cos 19\omega x - 0.014 \cos 21\omega x - 0.007 \cos 23\omega x \\ & + 0 + \text{etc.} \end{aligned}\right\} \quad (8)$$

The maximum stress σ_x belonging to each of the harmonics of this temperature distribution again can be taken from curve *A* of Fig. 4, and the summation of these stresses is performed in the same manner as before. As an example, consider the point $l/a = 3$ of Fig. 4. The stresses per unit temperature for the first, third, $\cdots$ etc. harmonics are taken from curve *A* as:

0.09 0.57 0.88 0.98 1.00 etc.

Thus the total stress is:

$$0.09 \times 0.254 + 0.57 \times 0.234 + 0.88 \times 0.200 + 0.98 \times 0.155 + 1 \times (\text{remainder} = 0.157) = 0.642.$$

This ordinate has to be plotted in Fig. 4 *not* at the point $l/a = 3$, because in our present temperature distribution l is only one-fifth of the fundamental half wave-length, so that it has to be plotted over the abscissa 3/5. The other points of curve *C*, Fig. 4, have been determined in the same manner.

APPENDIX II.

Temperature Stresses in Cylinders.

The general theory of the heat stresses in tubes with a radially symmetrical temperature distribution is worked out fully in Timoshenko and Lessells' "Applied Elasticity," pp.

146–147. Here this theory is applied to a number of cases having practical interest in the field of welding. The following notations are used:

x = distance along cylinder from origin.
y = radial displacement of a point on the cylinder toward center.
R = radius of cylinder.
t = thickness of material.
T = temperature (a function of x).
$I = t^3/12$ = bending moment of inertia of unit strip.

Then the differential equation of the problem is:

$$\frac{EI}{1-\nu^2}\cdot\frac{d^4y}{dx^4}+\frac{Et}{R^2}y=-\frac{\alpha ET}{R}t.$$

(*a*) *Sinusoidal Temperature along Tube.*—As a first temperature distribution, we consider a sinusoidal one along the *infinitely long tube* (Fig. 13*a*):

$$T=T_0\cos\omega x.$$

The complete solution of the differential equation for this case is:

$$y=e^{\beta x}[C_1\sin\beta x+C_2\cos\beta x]+e^{-\beta x}[C_3\sin\beta x+C_4\cos\beta x]-\frac{\alpha RT_0}{1+\frac{R^2t^2\omega^4}{12(1-\nu^2)}}\cdot\cos\omega x\cdots \quad (9)$$

where

$$\beta=\sqrt[4]{\frac{3(1-\nu^2)}{R^2t^2}}=\frac{2\pi}{\text{“ free wave-length ”}}.$$

Since in this case the tube extends to infinity both ways, one of the terms $e^{\beta x}$ or $e^{-\beta x}$ would become infinitely great at infinity, which is impossible. Thus we have for this simple case that all four C's are zero and that the deflection curve is a sine wave:

$$y=\frac{-\alpha RT_0}{1+\frac{R^2t^2\omega^4}{12(1-\nu^2)}}\cdot\cos\omega x.$$

The $-$ sign means that there is a negative inward displacement for a positive temperature, which is obviously correct.

The natural (unstressed) deflection toward the inside would be $-\alpha RT_0 \cos \omega x$ and the difference between this and the actual deflection y is felt as a tangential compressive stress σ_t:

$$\sigma_t = \frac{E}{R}(y + \alpha RT) = E\alpha T_0 \cos \omega x \frac{1}{1 + \frac{12(1 - \nu^2)}{R^2 t^2 \omega^4}}$$

$$= E\alpha T_0 \cdot \cos \omega x \frac{1}{1 + \frac{12(1 - \nu^2)}{\pi^4}(l^2/Rt)^2},$$

where l is half a wave-length: $\omega l = \pi$.

The fact that the tube is bent causes a bending stress:

$$\sigma_b = \frac{t/2}{I} \frac{EI}{1 - \nu^2} y'' = \frac{Et}{2(1 - \nu^2)} \cdot y''$$

$$= \alpha E T_0 \cos \omega x \cdot \frac{6/Rt\omega^2}{1 + \frac{12(1 - \nu^2)}{R^2 t^2 \omega^4}}$$

$$= \alpha E T_0 \cos \omega x \cdot \frac{\frac{6}{\pi^2}\frac{l^2}{Rt}}{1 + \frac{12(1 - \nu^2)}{\pi^4}(l^2/Rt)^2}.$$

The bending stress is a tension (or compression) in the longitudinal direction of the tube, while the tangential stress is along a circle. The maximum shear stress is half the sum (or difference) of the two and occurs at the points along the tube where $\cos \omega x = 1$,

$$\tau_{max.} = E\alpha T_0 \cos \omega x \frac{1 + \frac{6}{\pi^2} \cdot l^2/Rt}{1 + \frac{12(1 - \nu^2)}{\pi^4}(l^2/Rt)^2},$$

which, with the numerical value $\nu = 1/3.5$ for rolled steel, becomes:

$$\frac{\tau_{max.}}{E\alpha T_0} = \frac{1 + 0.61\, l^2/Rt}{1 + 0.114\, (l^2/Rt)^2}. \tag{10}$$

A plot of $\tau_{max.}/E\alpha T_0$ against $l/\sqrt{Rt}$ is shown in curve A, Fig. 14.

(*b*) *Tube of Finite Length with One-Half Temperature Sine Wave.*—For this case the same differential equation and the same general solution (9) with four integration constants applies. Taking the origin of coördinates in the center of the tube and the ends at $x = \pm l/2$, it is clear that the $y = f(x)$ diagram must be symmetrical with respect to the y-axis. Then we must have in the general solution $C_1 = -C_3$ and $C_2 = C_4$, with which the solution becomes:

$$y = C_1 \sin \beta x \sinh \beta x + C_2 \cos \beta x \cosh \beta x - \frac{\alpha R T_0}{1 + \dfrac{R^2 t^2 \omega^4}{12(1 - \nu^2)}} \cdot \cos \omega x.$$

The boundary conditions are that at the ends $x = \pm l/2$ there is no bending moment ($y'' = 0$) and no shear force ($y''' = 0$). Performing these differentiations we obtain two equations in the C's, which can be solved with the result:

$$C_1 = -\frac{1}{2} \frac{\alpha R T_0}{1 + \dfrac{R^2 t^2 \omega^4}{12(1 - \nu^2)}} \cdot \frac{\omega^3}{\beta^3} \frac{\sin \dfrac{\beta l}{2} \cdot \sinh \dfrac{\beta l}{2}}{\sin \dfrac{\beta l}{2} \cos \dfrac{\beta l}{2} + \sinh \dfrac{\beta l}{2} \cosh \dfrac{\beta l}{2}},$$

$$C_2 = -\frac{1}{2} \frac{\alpha R T_0}{1 + \dfrac{R^2 t^2 \omega^4}{12(1 - \nu^2)}} \cdot \frac{\omega^3}{\beta^3} \cdot \frac{\cos \dfrac{\beta l}{2} \cdot \cosh \dfrac{\beta l}{2}}{\sin \dfrac{\beta l}{2} \cos \dfrac{\beta l}{2} + \sinh \dfrac{\beta l}{2} \cdot \cosh \dfrac{\beta l}{2}}.$$

With these substituted in the expression for y, that deflection is calculated. The maximum shear stress is found from it as

before:

$$2\tau_{max.} = E\alpha T_0 + E\frac{y}{R} + \frac{Et}{2(1-\nu^2)}y''$$

$$= E\alpha T_0 \frac{1 + 0.61\, l^2/Rt}{1 + 0.114\,(l^2/Rt)^2} + \frac{E}{R}C_2 + \frac{Et}{\beta^2(1-\nu^2)}C_1, \quad (11a)$$

of which the first part is the solution (10) of the infinite tube found previously, and the second part is the correction due to the finiteness. Worked out this correction becomes:

$$\left(\frac{2\tau_{max.}}{E\alpha T_0}\right)_{corr.} = \frac{-\frac{1}{2}}{1 + \frac{R^2t^2\omega^4}{12(1-\nu^2)}} \cdot \frac{\omega^3}{\beta^3} \cdot \frac{\cos\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2} + \sqrt{\frac{3}{1-\nu^2}}\cdot\sin\frac{\beta l}{2}\cdot\sinh\frac{\beta l}{2}}{\sin\frac{\beta l}{2}\cdot\cos\frac{\beta l}{2} + \sinh\frac{\beta l}{2}\cdot\cosh\frac{\beta l}{2}}. \quad (11b)$$

With the formulæ (11a) and (11b) curve B of Fig. 14 has been calculated. It is seen that for large wave-lengths ($\omega \ll \beta$) the correction is negligible, and that only for comparatively short wave-lengths there is an appreciable difference. For very short wave-lengths the stress in the middle point of the tube is not the largest one, but the stress at the ends ($x = \pm l/2$) predominates. Since at these ends there is neither bending nor temperature rise, the stress is simply:

$$(2\tau)_{ends} = \frac{E}{R}y_{end},$$

or:

$$\left(\frac{2\tau}{E\alpha T_0}\right)_{ends} = \frac{-\frac{1}{2}}{1 + \frac{R^2t^2\omega^4}{12(1-\nu^2)}} \cdot \frac{\omega^3}{\beta^3} \cdot \frac{\cos^2\frac{\beta l}{2} + \sinh^2\frac{\beta l}{2}}{\sin\frac{\beta l}{2}\cdot\cos\frac{\beta l}{2} + \sinh\frac{\beta l}{2}\cosh\frac{\beta l}{2}}.$$

With the aid of this formula, the dotted branch to the left of point P in Fig. 14 was calculated. It may be checked that the expressions for the "end stress" as well as for the "correction stress" become $2/\pi$ for the limiting case $l = 0$, as explained in the main text.

(*c*) *Infinite Tube with Isolated Hot Half Sine Wave.*—Lay the origin in the center of the hot spot and consider the temperature distribution:

$$T = T_0 \cos(\pi x/l) \text{ for } -l/2 < x < l/2$$

and

$$T = 0 \qquad \text{for} \quad |x| > l/2.$$

Again the differential equation and the general solution of the previous cases apply. For the center portion the deformation y_1 has to be symmetrical, and for the end portions the deformation y_2 has to vanish at infinity, so that we have:

$$y_1 = C_1 \sin \beta x \sinh \beta x + C_2 \cos \beta x \cosh \beta x - \frac{\alpha R T_0}{1 + \frac{R^2 t^2 \omega^4}{12(1-\nu^2)}} \cdot \cos \omega x,$$

$$y_2 = e^{-\beta x}[C_3 \sin \beta x + C_4 \cos \beta x].$$

The four constants are determined from the fact that at the joint $x = l/2$ the y's and their first three derivatives must be the same, or:

$$y_1 = y_2,\ y_1' = y_2',\ y_1'' = y_2'',\ y_1''' = y_2''', \text{ at } x = l/2.$$

When the integration constants are solved from this and substituted in the expression for the shear stress, the result is as follows:

$$\begin{aligned}\frac{2\tau}{\alpha E T_0} = \cos \omega x &\left[1 - \frac{A}{\alpha R T_0} + \frac{A t \omega^2}{2\alpha T_0 (1-\nu^2)}\right] \\ &+ \frac{A}{\alpha R T_0}\left[\left(\frac{\omega^3}{2\beta^3} q - \frac{\omega}{\beta} p\right) \sin \beta x \sinh \beta x \right. \\ &\left. - \left(\frac{\omega^3}{2\beta^3} p + \frac{\omega}{\beta} q\right) \cos \beta x \cosh \beta x\right] \\ &+ \frac{A t \beta^2}{2\alpha T_0 (1-\nu^2)}\left[\left(\frac{\omega^3}{2\beta^3} q - \frac{\omega}{\beta} p\right) \cos \beta x \cosh \beta x \right. \\ &\left. + \left(\frac{\omega^3}{2\beta^3} p + \frac{\omega}{\beta} q\right) \sin \beta x \sinh \beta x\right],\end{aligned}$$

in which the following abbreviations occur:

$$A = \frac{\alpha R T_0}{1 + \frac{R^2 t^2 \omega^4}{12(1-\nu^2)}},$$

$$p = \frac{\begin{array}{r} + \sin\frac{\beta l}{2}\sinh\frac{\beta l}{2} + \cos\frac{\beta l}{2}\cosh\frac{\beta l}{2} \\ + \sin\frac{\beta l}{2}\cosh\frac{\beta l}{2} + \cos\frac{\beta l}{2}\sinh\frac{\beta l}{2} \end{array}}{2(\cosh\beta l + \sinh\beta l)},$$

$$q = \frac{\begin{array}{r} - \sin\frac{\beta l}{2}\sinh\frac{\beta l}{2} + \cos\frac{\beta l}{2}\cosh\frac{\beta l}{2} \\ - \sin\frac{\beta l}{2}\cosh\frac{\beta l}{2} + \cos\frac{\beta l}{2}\sinh\frac{\beta l}{2} \end{array}}{2(\cosh\beta l + \sinh\beta l)}.$$

The first line of the above solution is identical with that of the infinite tube with uniform sinusoidal distribution, whereas the next two terms represent the correction due to the cool ends. Since the stress is maximum for $x = 0$, we get for the correction at that point:

$$\left(\frac{2\tau}{\alpha E T_0}\right)_{corr.} = \frac{1}{1 + 8.8(Rt/l^2)^2}\left[-\frac{\omega^3}{2\beta^3}p - \frac{\omega}{\beta}q + \sqrt{\frac{3}{4(1-\nu^2)}}\left(\frac{\omega^3}{2\beta^3}q - \frac{\omega}{\beta}p\right)\right].$$

As in the previous case for long wave-lengths $\omega \ll \beta$, the expressions p and q become very small and with it the correction. For smaller wave-lengths the stress as determined by the above formula is shown by curve C of Fig. 14.

The same result can also be obtained by imposing on the tube the temperature distribution of Fig. 3c, which is expressed by the Fourier series (8). Operating with this series on curve A, Fig. 14, we arrive at curve C in exactly the same manner as was done with Fig. 4. Taking, for instance, $l/\sqrt{Rt} = 5$, the ordinates of curve A, Fig. 14, at that point

and at the points 5/3, 5/5, 5/7, etc. are seen to be respectively:

$$0.115, \quad 0.71, \quad 0.72, \quad 0.64, \cdots.$$

With the series (8) the new ordinate for the temperature distribution of Fig. 3*c* becomes:

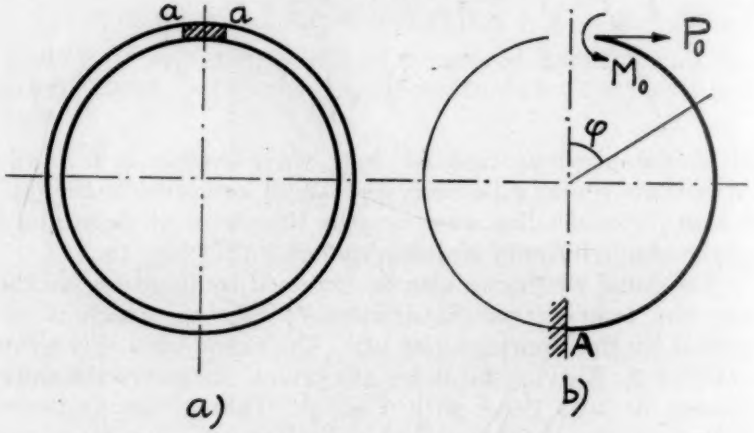
$$0.115 \times 0.254 + 0.71 \times 0.234 + 0.72 \times 0.200 + 0.64 \times 0.155 + 0.57 \times 0.109 + \text{approximately } 0.53 \times 0.048 = 0.524.$$

This ordinate has to be plotted over the abscissa $1/5 \times 5 = 1$, for the same reason as explained before in Appendix I, *c*.

(*d*) *Infinite Tube with Zig-Zag Temperature Distribution.*—The temperature distribution of Fig. 13*d* can be interpreted as a sum of sine waves by the series (4) as before. The stress in the tube caused by this distribution is obtained again by summing up the various ordinates of curve *A*, Fig. 14, resulting in curve *D*, Fig. 14.

(*e*) *Tube with Heated Longitudinal Seam.*—We want to show finally that the stress caused by bending and compression in the plane of a circular cross-section is very small in comparison to the tensile and compressive stress along the length of the cylinder with a heated seam. For this purpose, we consider in Fig. 17*a* a cold cylinder without stress. We

FIG. 17.



Calculation of bending stress in a tube with heated longitudinal seam.

take out a section aa of length l which is expanded by an amount $2\delta = \alpha T_0 l$ and then put it back into the ring. Let this cause a force P_0 and a moment M_0 at that point. The moment at φ then is:

$$M_\varphi = M_0 + P_0 R(1 - \cos \varphi)$$

and the energy in half the ring is:

$$E = \frac{R}{2EI}\int_0^\pi M^2 d\varphi.$$

There are two conditions: compared to the built-in section A the top of the ring (1) does not turn and (2) displaces to the right through a distance δ. Carrying out the calculations, we find

$$P_0 = M_0/R \quad \text{and} \quad M_0 = \frac{4EI\delta}{\pi R^2}$$

and it is also seen that these values are the highest ones anywhere along the circle.

Assume now that the heated seam aa has a width $l = R/5$. Then $\delta = R\alpha T_0/10$, and the maximum bending stress becomes $E\alpha T_0 \cdot t/5\pi R$. Assuming further that $R = 25t$, this stress comes out to be $E\alpha T_0/400$, which is negligible.

The compression stress is even smaller, being $t/6R$ times as large as the bending stress. With the same assumptions as before, this comes out to be $E\alpha T_0/60{,}000$.

And New Tea Seed Oil.—Apparently one phase of the work of the Food and Drug Administration is never finished, and that is catching up with the sellers of adulterated olive oil. In times past, cottonseed oil, peanut oil, Sesame oil, sunflower seed oil and others have served as adulterants. One after another the Federal chemists forced their abandonment and now have had to bestir themselves to find a test that would prove the presence of the cheap tea seed oil that is being substituted for the more expensive olive oil.

Tea seed oil is a commodity virtually unknown to most buyers of salad oils. It is pressed from the tea nut, or seed, produced by a plant closely related to that from which leaves for beverage tea are harvested. The major use of the oil, which is imported from the Orient, is in paint and textile industries. It may be used as a table oil when refined, and is not unwholesome. Tea seed oil resembles olive oil closely in many respects, and answers to all of the recognized tests for olive oil. Not until a new distinguishing test was developed could the Food and Drug Administration begin suit for the confiscation of the adulterated oil now on the market.

C.

BALANCING ROTORS BY MEANS OF ELECTRICAL NETWORKS.

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I. INTRODUCTION.

The rational methods in use for balancing rotors may be classified as (1) those which utilize a balancing machine in which the rotor is supported and rotated in a pivoted carriage [1] and (2) those in which the unbalance is determined by calculation from measurements of the vibration produced by the rotor's rotation.[2]

The disadvantages of (1) are: First, the rotor cannot be balanced in its own bearings. Second, the balancing speeds must be low, since at high speeds distortion in the carriage and consequent error in the unbalance indication is difficult to avoid. Third, unbalance indication for rotors requiring more than two correction locations cannot be made. The disadvantage of (2) is that the operator is required to make calculations which consume too much time for the quantity production of balanced rotors and which require training.

The disadvantages in the case of a balancing machine are associated with the pivoted carriage, the function of which is that of separating the two unbalance effects. It is natural therefore to consider other means for performing this function. Since electrical methods of measuring vibration are convenient, it is reasonable to consider the possibility of using electrical means of separating the unbalance effects. It turns out that electrical networks can be used for this purpose and that such means avoids the disadvantages of (2) as well as those of the pivoted carriage (1). A rotor may be balanced

[1] Akimoff, *Trans. A. S. M. E.*, **38** (1916); Soderberg, C. R., *Trans. A. S. M. E.*, **45** (1923); Thearle, E. L., *Trans. A. S. M. E.*, **54** (1932); Tinius Olsen Company's "Vibro Electric Balancing Machine" exhibited at Century of Progress 1933.

[2] Rathbone, T. G., "Turbine Vibration and Balancing," *Trans. A. S. M. E.*, 1929; Thearle, E. L., "Dynamic Balancing of Rotating Machinery in the Field," *Trans. A. S. M. E.*, 1934.

at practically any speed without calculation or special skill on the part of the operator in its own bearings or in almost any mechanically suitable bearing arrangement. The possibility of high balancing speeds for high speed rotors is important for rotors in which non-symmetrical distortion may occur due to fast rotation.

Definition of Balance-Unbalance Correction.

By balancing is meant the process in which a rotor's mass distribution is altered to reduce vibration at arbitrary points in the rotor's supporting structure or to reduce vibrations of its shaft.

In the most general case it can be shown that there must be as many arbitrary locations along (but of course not on) the axis of rotation in which unbalance corrections may be made as there are locations at which it is required to eliminate vibration. However, it is usually possible, for a two bearing rotor to choose two locations along the rotor's axis where the proper corrections will eliminate practically all vibration in the rotor supporting structure and on its shaft for at least one speed of rotation. Furthermore, for rotors of three or more bearings, usually all that is required is two correction locations for rotors with a rigid supporting structure between bearings and one correction location per bearing for rotors with flexible structures between bearings.

II. ANALYTICAL BASIS OF THE USE OF A NETWORK FOR BALANCING.

Figure 1 represents a perfectly balanced rotor supported in the bearing pedestals a and b and rotating at a constant speed. Let this system be linear, i.e., a force of a given frequency acting at a given point in a given direction will produce motions which are proportional to this force.

If unbalances w_1 and w_2 (vectors) are inserted in planes 1 and 2 respectively, let the vectors x_a and x_b be such that their projections are the vibratory displacements in one direction at a and b respectively due to w_1 and w_2. Since the system is linear, the rotor symmetrical, and perfectly in balance without w_1 or w_2, x_a may be divided into the components x_{a_1} due to the unbalance w_1 and x_{a_2} due to unbalance w_2, and x_b may be divided similarly into x_{b_1} and x_{b_2}.

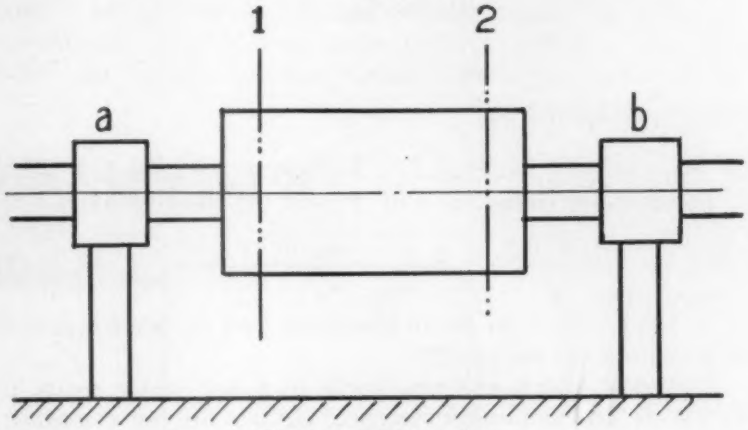
Thus

$$x_a = x_{a_1} + x_{a_2}, \tag{1}$$

$$x_b = x_{b_1} + x_{b_2}. \tag{2}$$

Let any Greek letter, for example β, with any subscript, be an operator such that its product with a vector, for example, x, results in a vector βx of a certain angular shift and a certain ratio of change in length with respect to x.

FIG. 1.



Sketch of two bearing rotor.

x_{a_1} will have a fixed angular relation and a fixed ratio of length to w_1. Thus

$$x_{a_1} = \alpha_{a_1} w_1 \tag{3}$$

and

$$x_{b_1} = \alpha_{b_1} w_1. \tag{4}$$

Similarly

$$x_{a_2} = \alpha_{a_2} w_2 \tag{5}$$

and

$$x_{b_2} = \alpha_{b_2} w_2. \tag{6}$$

Elimination of w_1 from (3) and (4) gives

$$x_{a_1} = \frac{\alpha_{a_1}}{\alpha_{b_1}} x_{b_1}, \tag{7}$$

which means that x_{a_1} and x_{b_1} have a fixed ratio of length and a fixed angular relation as indicated by the operator $\alpha_{a_1}/\alpha_{b_1}$.

In order to indicate the unbalance w_2 we seek a combination of x_a and x_b which will depend only on w_2. Inspection of (1) and (2) indicates that $x_a + \beta_2 x_b$ depends only on w_2, if β_2 is such that $x_{a_1} + \beta_2 x_{b_1} = 0$.

In short, if

$$x_{a_1} + \beta_2 x_{b_1} = 0, \tag{8}$$

$$x_a + \beta_2 x_b = x_{a_2} + \beta_2 x_{b_2}, \tag{9}$$

and from (5) and (6)

$$x_{a_2} + \beta_2 x_{b_2} = (\alpha_{a_2} + \beta_2 \alpha_{b_2}) w_2 \tag{10}$$

or

$$x_a + \beta_2 x_b = \gamma_2 w_2, \tag{11}$$

where γ_2 is defined by

$$\gamma_2 = (\alpha_{a_2} + \beta_2 \alpha_{b_2}). \tag{12}$$

Incidentally comparison of (7) and (8) shows that

$$\beta_2 = -\frac{\alpha_{a_1}}{\alpha_{b_1}}. \tag{13}$$

(11) is the basis for the use of a network and (8) is the criterion for adjusting the network.

Metering and operations such as β are easily made in electric circuits (although electrical circuits are not essential). For practical ends then the motions at points a and b are converted by vibration pick-ups into voltages to correspond to the vectors E_a and E_b respectively. Assuming the two pick-ups are identical (which is not necessary for the operation of the scheme) the relation between E_a and x_a is

$$E_a = \varphi x_a. \tag{14}$$

That between E_b and x_b is

$$E_b = \varphi x_b, \tag{15}$$

in which φ depends only on the electrical characteristics of the vibration pick-ups. Defining E_{a_1}, E_{a_2}, E_{b_1}, and E_{b_2} by

$$\left.\begin{aligned} E_{a_1} &= \varphi x_{a_1} \\ E_{b_1} &= \varphi x_{b_1} \end{aligned}\right\}, \tag{16}$$

$$\left.\begin{aligned} E_{a_2} &= \varphi x_{a_2} \\ E_{b_2} &= \varphi x_{b_2} \end{aligned}\right\}. \tag{17}$$

Substituting (16) in (8)

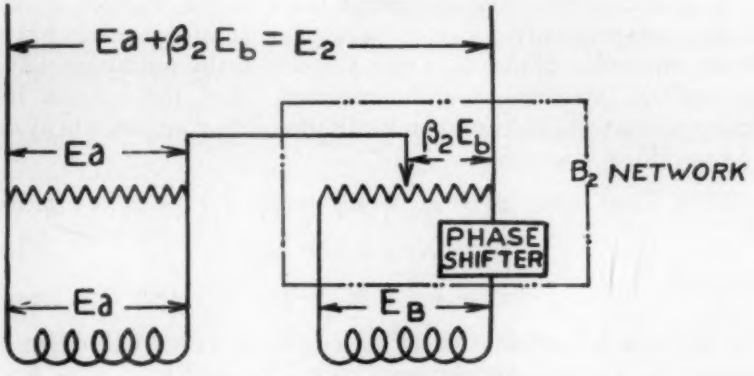
$$E_{a_1} + \beta_2 E_{b_1} = 0 \tag{8.1}$$

and substituting (14) and (15) in (11)

$$E_a + \beta_2 E_b = \varphi\gamma_2 w_2 = E_2. \tag{11.1}$$

(11.1) indicates that if an electrical network B_2, which performs the operation β_2 is used for shifting phase and changing amount of the voltage E_b the resulting voltage $\beta_2 E_b$ when placed in series with E_a will be a voltage $\varphi\gamma_2 w_2 = E_2$ which is proportional to and in fixed phase relation with w_2. (8.1) indicates that the network B_2 (Fig. 2) is adjusted properly when

FIG. 2.



Network for two bearing rotor.

an unbalance w_1 *only* results in $E_a(= E_{a_1})$ and $\beta_2 E_b(= \beta_2 E_{b_1})$ adding to zero. Once B_2 is adjusted the proportionality factor and the phase relation between E_2 and w_2 represented by the operator $\varphi\gamma_2$ can be determined by inserting a known w_2.

Similarly with the same two pick-up voltages E_a and E_b a network B_1 may be adjusted so that its operation β_1 results in

$$E_{a_1} + \beta_1 E_{b_1} = 0, \tag{8.2}$$

under which condition,

$$E_a + \beta_1 E_b = \varphi\gamma_1 w_1 = E_1. \tag{11.2}$$

The above adjustments of the networks B_1 and B_2 and the calibration to determine $\varphi\gamma_1$ and $\varphi\gamma_2$ are described with a

perfectly balanced rotor. If a perfectly balanced rotor is not available the effects on the pick-up voltages at a and b due to initial unbalance may be removed by inserting equal and opposite voltages in series with the pick-ups. In this way a perfectly balanced rotor may be simulated in the electrical circuit. On completion of the adjustments of B_1 and B_2 and the calibration to determine $\varphi\gamma_1$ and $\varphi\gamma_2$ the inserted voltages may be removed and the resulting E_1 and E_2 interpreted by means of the known $\varphi\gamma_1$ and $\varphi\gamma_2$ as the unbalance w_1 and w_2 which will be the actual total unbalances of the machine, or more strictly the weights which must be removed or compensated for if the motions at a and b are to be made zero.

For the case of three correction planes let the motions at locations a, b and c be the projections of the vectors x_a, x_b and x_c respectively. Let a perfectly balanced rotor have three correction planes 1, 2 and 3 in which the unbalances w_1, w_2 and w_3 (vectors) may be inserted. Let the system be linear so that the motions can be divided into components as in the two location case.

$$x_a = x_{a_1} + x_{a_2} + x_{a_3}, \tag{1.0}$$

$$x_b = x_{b_1} + x_{b_2} + x_{b_3}, \tag{2.0}$$

$$x_c = x_{c_1} + x_{c_2} + x_{c_3}. \tag{3.0}$$

We seek a combination of x_a, x_b and x_c which will depend only on w_1, i.e., will depend only on x_{a_1}, x_{b_1}, and x_{c_1}. $x_a + \delta_1 x_b$ will be independent of w_2 if δ_1 is defined by

$$x_{a_2} + \delta_1 x_{b_2} = 0. \tag{4.0}$$

Thus adding (1.0) to δ_1 times (2.0) and making use of (4.0)

$$x_a + \delta_1 x_b = x_{a_1} + \delta_1 x_{b_1} + x_{a_3} + \delta_1 x_{b_3}. \tag{5.0}$$

$x_c + \psi_1 x_b$ will also be independent of w_2 if ψ_1 is defined by

$$x_{c_2} + \psi_1 x_{b_2} = 0. \tag{6.0}$$

Thus adding (3.0) to ψ_1 times (2.0) and making use of (6.0)

$$x_c + \psi_1 x_b = x_{c_1} + \psi_1 x_{b_1} + x_{c_3} + \psi_1 x_{b_3}. \tag{7.0}$$

The vector $(x_{a_3} + \delta_1 x_{b_3})$ of (5.0) and the vector $(x_{c_3} + \psi_1 x_{b_3})$ of (7.0) both depend on w_3 alone and therefore are in a fixed

phase relation with respect to each other and are of a fixed ratio of length. The sum of $x_a + \delta_1 x_b$ from (5.0) and $\eta_1(x_c + \psi_1 x_b)$ of (7.0) is independent of w_3 if η_1 is defined by

$$x_{a_3} + \delta_1 x_{b_3} + \eta_1(x_{c_3} + \psi_1 x_{b_3}) = 0. \tag{8.0}$$

Thus adding (5.0) to η_1 times (7.0) and making use of (8.0)

$$x_a + \delta_1 x_b + \eta_1(x_c + \psi_1 x_b) = x_{a_1} + \delta_1 x_{b_1} + \eta_1(x_{c_1} + \psi_1 x_{b_1}). \tag{9.0}$$

The right hand member of (9.0) is a function only of w_1 so that (9.0) may be written:

$$x_a + \delta_1 x_b + \eta_1(x_c + \psi_1 x_b) = \lambda_1 w_1. \tag{10.0}$$

(10.0) is the basis for a balancing machine for three balancing planes. The equation corresponding to (10.0) for w_2 and w_3 of course are similar.

Pick-ups are used at points a, b and c to convert the motions into voltages corresponding to the vectors E_a, E_b and E_c respectively. Assuming that the three pick-ups are identical the electrical relations similar to equations (4.0) to (10.0) inclusive are:

$$E_{a_3} + \delta_1 E_{b_3} = 0, \tag{4.01}$$

$$E_a + \delta_1 E_b = E_{a_1} + \delta_1 E_{b_1} + E_{a_3} + \delta_1 E_{b_3}, \tag{5.01}$$

$$E_{c_3} + \psi_1 E_{b_3} = 0, \tag{6.01}$$

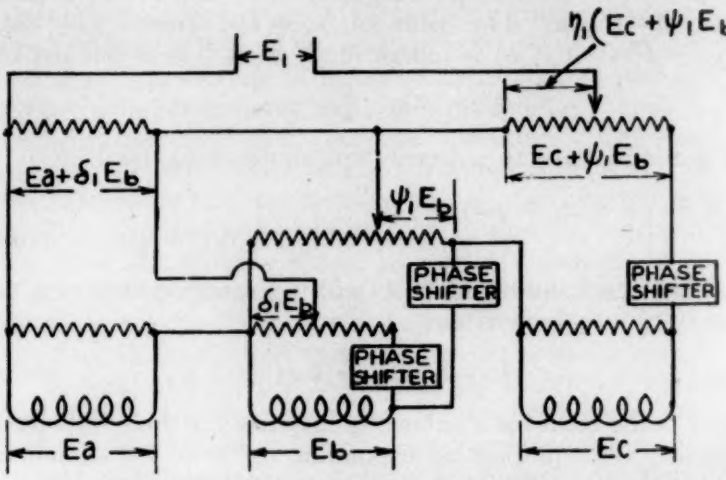
$$E_c + \psi_1 E_b = E_{c_1} + \psi_1 E_{b_1} + E_{c_3} + \psi_1 E_{b_3}, \tag{7.01}$$

$$E_{a_3} + \delta_1 E_{b_3} + \eta_1(E_{c_3} + \psi_1 E_{b_3}) = 0, \tag{8.01}$$

$$E_a + \delta_1 E_b + \eta_1(E_c + \psi_1 E_b) = \varphi\lambda_1 w_1 = E_1. \tag{10.01}$$

Figure 3 represents a network corresponding to equation (10.01). With a perfectly balanced rotor or a rotor where perfect balance is simulated in the electrical circuit by placing an equal and opposite voltage in series with the initial voltage in each pick-up, equations (4.01), (6.01) and (8.01) are the basis for the adjustment of the network which is carried out as follows (note Fig. 3):

FIG. 3.



Network for three bearing rotor.

(1) Place an unbalance w_2 *only* in the rotor.

(2) With a phase shifter and a potentiometer perform the operation δ_1 on E_b to get

$$E_a(=E_{a_2}) + \delta_1 E_b(=\delta_1 E_{b_2}) = 0 \text{ (eq. 4.01)}.$$

(3) Under the same conditions of running perform the operation ψ_1 on E_b to get

$$E_c(=E_{c_2}) + \psi_1 E_b(=\psi_1 E_{b_2}) = 0 \text{ (eq. 6.01)}.$$

(4) Place an unbalance w_3 in the rotor either leaving w_2 in the rotor or removing it.

(5) With a phase shifter and a potentiometer perform the operation

$$\eta_1 \text{ on } (E_c + \psi_1 E_b)$$

to get $E_a + \delta_1 E_b + \eta_1(E_c + \psi_1 E_b) = 0$ (eq. 8.01).

(6) Once this network is adjusted the proportionality factor and the phase relation $\varphi\lambda_1$ between the output of the network, E_1, and w_1 can be obtained by inserting a known w_1.

In a similar manner networks for determining w_2 and w_3 can be adjusted. This same procedure is applicable to a

system having n planes for unbalance correction where the motion is to be brought to zero at all of n points in a single direction at each point.

In cases where rotors are balanced in their own bearings, experience has shown that the motion responses are often only approximately linear with respect to the forces. Insofar as there is this non-linearity the method described does not give exact determinations, with the result that after the corrections for the unbalances as indicated are carried out repeat determination and subsequent further correction of unbalances may be necessary. The calibration and setting of *network* constants used for the first determination will probably be suitable for the second.

III. METHODS OF INDICATION.

Once the network has produced a voltage proportional to the unbalance in one correction plane and in fixed phase relation with it, the problem becomes simply that of measuring the amount of the voltage and phase of the voltage with respect to the rotation of the rotor. These measurements can be made in several ways, as for example:

1. The amount of voltage is measured with an ordinary a.c. meter calibrated in ounces of unbalance. The phase of the voltage, which corresponds to the position of unbalance correction is indicated by having it control the flashing of a stroboscopic light. A stationary pointer is mounted so that the stroboscopic light illuminates the rotor in that angular position in which the unbalance is opposite the pointer. The angular position of the unbalance is recognized by means of numbers or other identification marks printed on or attached to the rotor. This scheme is well suited for quantity balancing of rotors.

2. This measurement of phase and amount of a voltage is made by using an indicating a.c. wattmeter together with a small two pole sine wave generator direct-coupled or geared, 1 : 1, to the rotor being balanced. The frequency of the generated voltage is therefore exactly the same as the frequency of the voltage in the network corresponding to the unbalance. By placing the network voltage on one coil and the generator voltage on the other coil of the wattmeter, an

indication proportional to the product of the two voltages multiplied by the cosine of the angle between them is obtained. The generator is built with an adjustable stator; and turning this stator until the meter reads zero (where the cosine of the angle between the voltages in the two-coils is 0) and observing the position of the stator which is marked in degrees gives a sharp indication of the phase of the network voltage. Then shifting the phase of the generator 90° in either direction gives a maximum deflection of the meter, which is in proportion to the network output as long as the amplitude of the generator output is held constant.

An important advantage of this scheme is that other vibration frequencies than that due to the unbalance which may be present in an actual case do not affect the wattmeter reading. This follows from the well known fact that the average product of two sine waves of different frequencies is zero. Sometimes it is desirable to balance machines on the bases of the motion of the shaft. In such a case this filtering feature of the wattmeter scheme is very helpful in eliminating the errors in the unbalance indication due to rough shaft surfaces. This method of voltage measurement is being applied to the balancing of rotors in their own pedestals in the field as well as on production balancing machines for large rotors.[3]

3. A conventional a.c. potentiometer is used to measure the phase and amount of the network voltage.

4. The network voltage is placed by means of collector rings on the segments of a two segment commutator mounted on a shaft direct coupled or geared 1 : 1 to the rotor shaft. Angularly adjustable brushes are used on the commutator. A d.c. ammeter is connected across these brushes. The phase of the voltage to be measured is indicated by the angular position of the brushes when they are adjusted to give zero meter readings. When the brushes are moved 90° from this position the meter reading gives the amount of the voltage.[4]

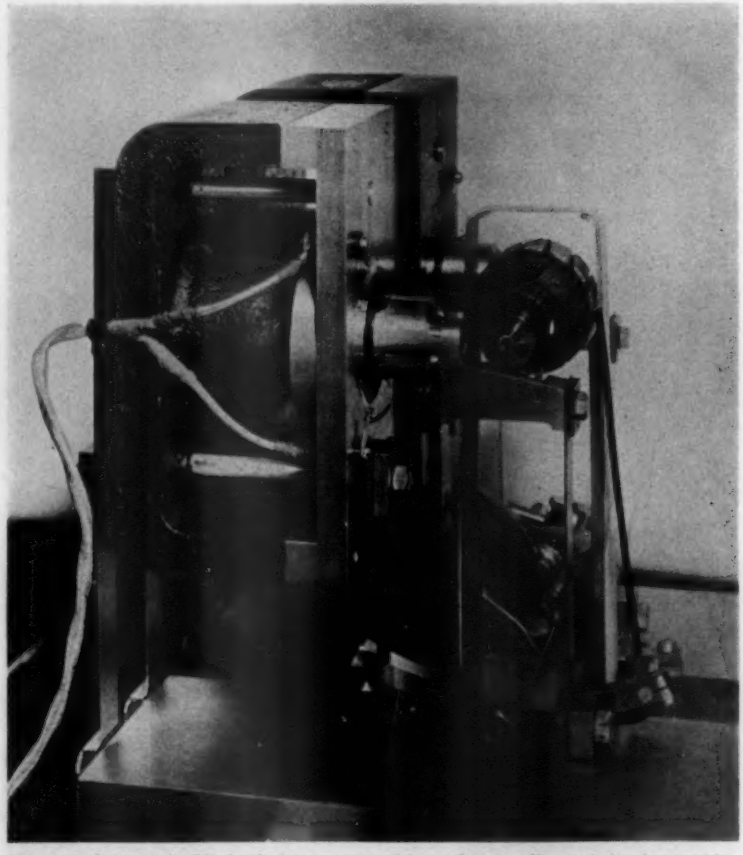
[3] This scheme of field balancing requires no calculations. Methods using measurements of phase and amount of motion at two points along the rotor axis together with vector calculations have been used by T. C. Rathbone (loc. cit., 2) and E. L. Thearle (loc. cit., 2).

[4] To the best of the authors' knowledge the first use of the commutator method for measuring phase and amount of pedestal vibration as an approximate indication of unbalance was made by Mr. Burk of the Burk Electric Company in 1915.

IV. DESCRIPTION OF TYPICAL MACHINE.

Figures 4 and 5 show respectively a detail and the general arrangement of a typical machine. The flexible mounting for

FIG. 4.

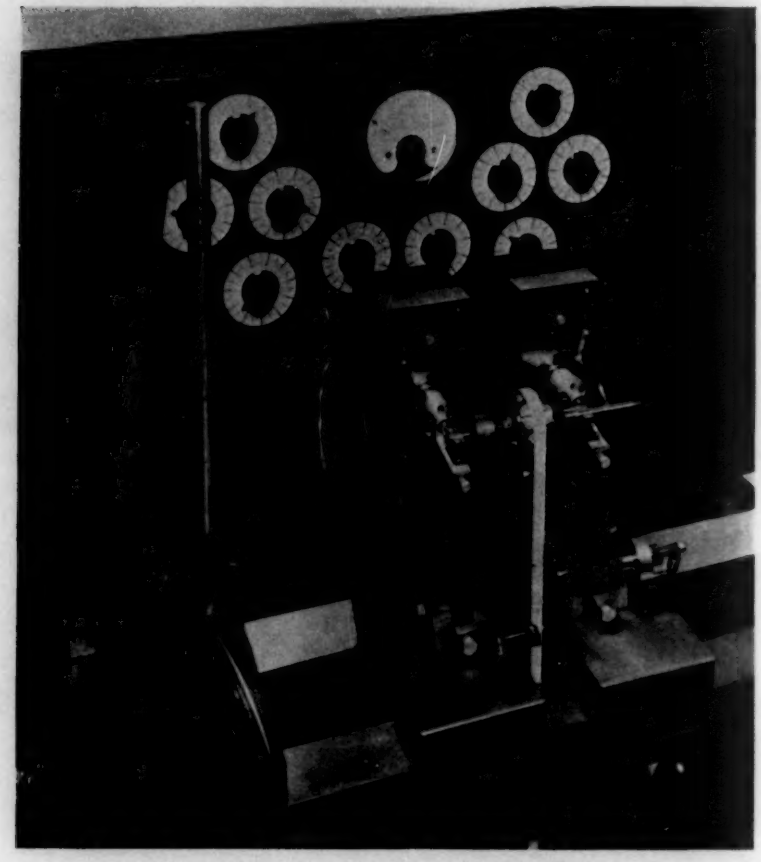


(Photograph 225984.) Mechanical set-up with pick-ups for a small rotor balancing machine.

A pivoted carriage type of balancing machine using the commutator method for measuring the phase and amount of unbalance was exhibited at the Century of Progress, 1933. It is the authors' understanding that Mr. Ohlson of the Tinius Olsen Company is responsible for this development. A similar method, using a cam and contactor commutator instead of a conventional commutator for obtaining vibration measurements to be used in the calculation of unbalance corrections was recently described by Mr. E. L. Thearle (loc. cit., 2).

the rotor consists simply of two bearings, each mounted for flexibility in a horizontal direction only. This flexibility is obtained by use of two vertical spring steel strips to support each bearing as shown in Fig. 4.

FIG. 5.



(Photograph 225987.) Balancing machine for small rotors.

The large *B* shaped parts to the left of the rotor in Fig. 4 are two permanent magnets. These magnets, together with the coils which are wound on the cylindrical pieces attached to the bearing supports and extending into the air gaps of the magnets, form the magnetic pick-ups. Thus the two points

along the axis of the rotor at which the vibratory motion is to be measured are chosen in this case at the bearing pedestals.

The magnet mountings are similar to those of the bearings. This combination of mountings eliminates the effect of floor vibration.

The cabinet shown in Fig. 5 contains the network, the necessary power supply and control for the stroboscopic lamp mounted in a reflector to the left, a circuit and a meter for measuring amount of unbalance, and switches necessary for convenient operation. There are three of these operation switches, as are shown on the right of the cabinet panel. The double switch on the extreme right is an "end switch" which selects one or the other of the two networks for operation. Thus with the switch at the "right end" position the indications (meter and stroboscope) apply to the right end correction plane; with the switch at the "left end" position the indications apply to the left end. The center switch is an "angle-amount switch" which places the output of whichever network is chosen by the "end switch" either into the a.c. meter circuit or into the stroboscopic lamp circuit. The switch to the left of the group is a "sensitivity switch" with which either of two meter scales of different sensitivities can be chosen.

The dials shown on the cabinet are for use in adjusting the machine for a particular type of rotor.

The rotor is rotated by a belt and a motor as shown in Fig. 5. For facility in placing and removing the rotor a belt lifter mechanism is provided such as is shown in the lifted position in Fig. 4.

V. SENSITIVITY AND ACCURACY OF MACHINE DESCRIBED.

A minimum unbalance of an amount sufficient to cause the bearing pedestals to vibrate at 0.00005 inch amplitude can be indicated. This is of the order of 0.001 ounce inch unbalance at one correction plane of a 2 inch diameter motor rotor. The accuracy of unbalance indication for unbalances of the order of 0.05 ounce inch in a 2 inch diameter motor rotor is within 3% error.

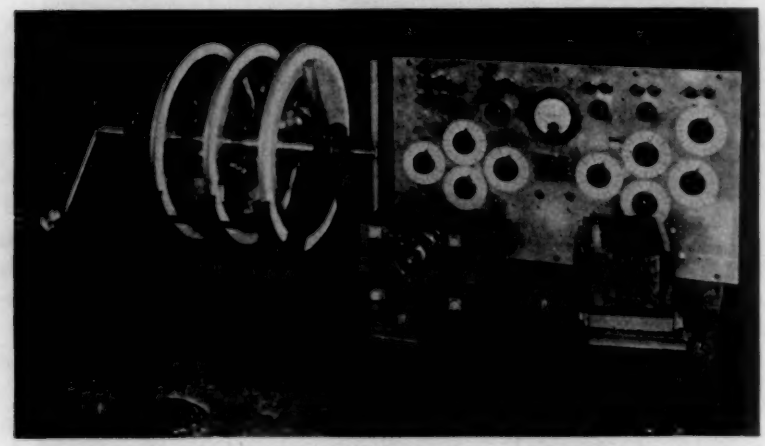
VI. TYPICAL APPLICATIONS.

Successful applications of this scheme have been made for small rotors from 1¼ inch diameter up to 3 inch diameter; me-

dium sized rotors up to 100 lb. in weight; and for very large rotors, such as turbo-generator rotors. In all cases the speeds of balancing in use are from 1,200 to 1,800 r.p.m. with one exception in which the balancing speeds are from 6,000–8,000 r.p.m.

In the high speed case the rotor is mounted in bearings similar to its own. In the balancing of very large rotors in their own bearings the apparatus of Fig. 6 [5] is used. For shop production of small and medium sized rotors the mountings shown in Fig. 5 [6] are being used.

FIG. 6.



(Photograph 227571.) Field balancing equipment showing two pick-ups, three a.c. generators, and the switch panel including network for a two bearing balancing equipment.

ACKNOWLEDGMENTS.

This balancing development has been carried out at the Westinghouse Research Laboratories. The help and suggestions of Messrs. L. E. Swedlund, W. O. Osbon, W. E. Bahls, C. R. Hanna, and S. L. Burgwin on the electric circuit problems, Mr. C. S. Williams on the magnetic problems, and of Messrs. H. C. Werner and B. Cametti on mechanical problems have been most essential. The authors appreciate this coöperation.

[5] This apparatus was used with good results on May 11, 1935 in balancing the largest turbo-generator rotor ever constructed, weight 250,000 pounds.

[6] This machine has been in use for production since December 1934.

CIRCULAR LOCI OF CURRENTS AND VOLTAGES IN A GENERAL NETWORK.

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SYNOPSIS.

A study is made of the expressions representing currents and voltages in a general network at fixed frequency. It is shown that both quantities follow circular loci when any one self impedance is varied.

INTRODUCTION.

Circular loci traversed by currents and voltages in certain types of networks, when an impedance element is varied along a straight line in the complex plane have been studied by many investigators. The facility with which the circle can be determined from three known conditions and the ease with which any point on the circular locus can be identified with the corresponding value of the variable, has brought about extended use of the circle diagram in performance calculations of synchronous and asynchronous machinery, power transmission lines, radio frequency transmission lines, transformers, circuits involving motional impedance and bridge circuits.

That the variation of impedance along a straight line in a simple series or parallel circuit produces a circular locus of current and voltage is demonstrated in many textbooks on alternating currents. In a number of studies made by the author on various types of constant frequency circuits, the presence of circular loci for all currents and voltages when self impedances were varied, made desirable a rigorous formulation of the conditions which engendered circular loci in a general circuit.

It will be shown that when a self impedance is varied in a general circuit, any current or voltage in the system may be expressed as a quantity having a circular locus. For this purpose, it will be found convenient to treat the circle as a linear fractional transformation, as originally applied to

alternating current problems by Schenkel.[1] The Schenkel form is

$$S = \frac{\alpha + \beta\rho}{\gamma + \delta\rho}$$

in which S is a vector describing a circle as the scalar variable, ρ, ranges from minus to plus infinity and α, β, γ and δ are complex constants. The radius and the center vector may be evaluated in terms of these constants using expressions developed by Schumann.[2] Generally, however, it will be found more expedient to determine the invariant points of the circle, namely, when $\rho = 0$ and when $\rho = \infty$ and use for the third point any convenient value of ρ. These three points not only suffice to locate the circle but also to fix the linear scale line which connects points on the circle with corresponding values of the variable ρ. With the scale line drawn, the value of S may be read off the plot in magnitude and phase angle for any value, positive or negative, of the variable scalar. The construction of the circle and the linear scale line has been described in a previous paper.[3]

THE GENERAL NETWORK.

Any network can be described by the values of the self and mutual impedance elements, the configuration of the junction points of the impedances and the sources of electromotive force acting in the system. All impedances will be considered as linear and bilateral. Suppose there are s impedance elements joined together with p junctions. The least number of meshes in the circuit will be n, given by [4]

$$n = s - p + 1.$$

Let the meshes be numbered from 1 to n and let the resultant electromotive forces acting in the meshes be E_1, $E_2 \cdots E_n$. These electromotive forces will be assumed to be sinusoidal, constant and of the same frequency. Using Maxwell's circulating currents, the mesh currents are labelled I_1, $I_2 \cdots I_n$. By applying Kirchhoff's and Ohm's Laws to each of the n meshes, n equations are obtained, which are:

$$
\begin{aligned}
I_1A_{11} + I_2A_{12} + I_3A_{13} + \cdots I_nA_{1n} &= E_1,\\
I_1A_{21} + I_2A_{22} + I_3A_{23} + \cdots I_nA_{2n} &= E_2,\\
I_1A_{31} + I_2A_{32} + I_3A_{33} + \cdots I_nA_{3n} &= E_3,\\
\cdots\cdots\cdots

Using Cramer's Rule the solution of any mesh current as I_k may be written as

$$I_k = \frac{\Delta_{I_k}}{\Delta},$$

in which Δ is the general determinant of the system of equations and Δ_{I_k} is the specific determinant for I_k, that is, the determinant obtained by replacing the kth column of A's with the column of E's.

The problem now is to examine the manner in which the self and mutual impedance elements enter into the various terms of Δ and of Δ_{I_k}. If the self impedance terms occur only to the first power in Δ and Δ_{I_k} then any mesh current can be written as a linear fractional transformation with any one self impedance regarded as a variable. The general determinant Δ is

$$\Delta = \begin{vmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & \cdots & A_{1n} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} & \cdots & A_{2n} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} & \cdots & A_{3n} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} & \cdots & A_{4n} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} & \cdots & A_{5n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{n1} & A_{n2} & A_{n3} & A_{n4} & A_{n5} & \cdots & A_{nn} \end{vmatrix}.$$

The principal diagonal consists of mesh impedance terms only. Symmetrically disposed about the principal diagonal are like coefficients, A_{21} A_{12}, A_{52} A_{25}, etc., which are equal because the impedances are bilateral. If any two meshes be considered all the impedance elements common to these meshes will be found in the minors containing only the subscripts of these meshes. Such minors are of rank two and a few of them are

$$M_{12} = \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix}, \quad M_{25} = \begin{vmatrix} A_{22} & A_{25} \\ A_{52} & A_{55} \end{vmatrix}, \quad M_{1n} = \begin{vmatrix} A_{11} & A_{1n} \\ A_{n1} & A_{nn} \end{vmatrix}.$$

It follows from La Place's development of a determinant, that if M is any minor of a determinant Δ, the product of M by its algebraic complement will produce, upon expansion, some of the terms of the expansion of Δ.[5] The complement of a minor is the matrix obtained by striking out the rows and

columns of the minor from the original determinant. The algebraic complement is the complement with a plus or minus sign according to whether the sum of the numbers indicating the rows and columns of the minor is even or odd, respectively. That is, if C_{kp} is the complement of M_{kp}, the algebraic complement is $C_{kp} \times (-1)^{k+p+k+p}$. Each of the minors written out above, multiplied by its algebraic complement, then will yield some terms of the expansion of the determinant.

For example, suppose the minor involving all impedance elements common to the kth and pth meshes be chosen. This minor with its algebraic complement may be written as

$$M_{kp}C_{kp}(-1)^{k+p+k+p},$$

which is

$$\begin{vmatrix} A_{kk} & A_{kp} \\ A_{pk} & A_{pp} \end{vmatrix} C_{kp}(-1)^{k+p+k+p}.$$

The matrix of the complement C_{kp} for the general circuit is as follows:

$$C_{kp} = \begin{vmatrix} A_{11} & A_{12}\cdots A_{1,\,k-1} & A_{1,\,k+1}\cdots A_{1,\,p-1} & A_{1,\,p+1}\cdots A_{1n} \\ A_{21} & A_{22}\cdots A_{2,\,k-1} & A_{2,\,k+1}\cdots A_{2,\,p-1} & A_{2,\,p+1}\cdots A_{2n} \\ A_{31} & A_{32}\cdots A_{3,\,k-1} & A_{3,\,k+1}\cdots A_{3,\,p-1} & A_{3,\,p+1}\cdots A_{3n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{k-1,\,1} & A_{k-1,\,2}\cdots A_{k-1,\,k-1} & A_{k-1,\,k+1}\cdots A_{k-1,\,p-1} & A_{k-1,\,p+1}\cdots A_{k-1,\,n} \\ A_{k+1,\,1} & A_{k+1,\,2}\cdots A_{k+1,\,k-1} & A_{k+1,\,k+1}\cdots A_{k+1,\,p-1} & A_{k+1,\,p+1}\cdots A_{k+1,\,n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{p-1,\,1} & A_{p-1,\,2}\cdots A_{p-1,\,k-1} & A_{p-1,\,k+1}\cdots A_{p-1,\,p-1} & A_{p-1,\,p+1}\cdots A_{p-1,\,n} \\ A_{p+1,\,1} & A_{p+1,\,2}\cdots A_{p+1,\,k-1} & A_{p+1,\,k+1}\cdots A_{p+1,\,p-1} & A_{p+1,\,p+1}\cdots A_{p+1,\,n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n1} & A_{n2}\cdots A_{n,\,k-1} & A_{n,\,k+1}\cdots A_{n,\,p-1} & A_{n,\,p+1}\cdots A_{nn} \end{vmatrix}.$$

No impedance elements common to the kth and pth meshes are present in the complement C_{kp}. All impedance elements common to the kth and pth meshes are in the minor M_{kp} and hence will be multiplied by the common factor, $C_{kp}(-1)^{k+p+k+p}$. Therefore to observe the effects of the impedances common to the kth and pth meshes, in as far as their relationships are concerned in the expansion of Δ, it is only necessary to examine the minor M_{kp}. The principal diagonal of this minor consists of the mesh impedances of the two meshes and a portion of these impedances may be common to the two meshes. The other diagonal of the minor consists only of the common impedances between the two meshes. Since the expansion of

the minor is

$$M_{kp} = A_{kk}A_{pp} - (A_{kp})^2,$$

the squares of the common self impedance elements will cancel out leaving only first powers of self impedance elements, mixed products of self and mutual impedances and squares of mutual impedances. This can be illustrated by expanding the minor M_{kp} as applied to the section of the general network in Fig. 1. Thus

$$\begin{vmatrix} A_{kk} & A_{kp} \\ A_{pk} & A_{pp} \end{vmatrix} = \begin{vmatrix} Z_1 + Z_2 + Z_3 + 2Z_{12} + 2Z_{23} + 2Z_{31} & -\left[Z_3 + Z_{13} + Z_{14} + Z_{23} + Z_{24} + Z_{34} \right] \\ -\left[Z_3 + Z_{13} + Z_{14} + Z_{23} + Z_{24} + Z_{34} \right] & Z_3 + Z_4 + 2Z_{34} \end{vmatrix} =$$

$$= Z_3(Z_1 + Z_2 + Z_4 + 2Z_{12} - 2Z_{14} - 2Z_{24}) + (Z_4 + 2Z_{34})(Z_1 + Z_2 + 2Z_{12} + 2Z_{23} + 2Z_{31}) - (Z_{13} + Z_{14} + Z_{23} + Z_{24} + Z_{34})^2$$

The square of the common self impedance Z_3, has cancelled out in this expansion. This process may be repeated and the complement C_{kp} broken down into other two-rank minors with their sub-complements, choosing the minors so that they involve all the impedance elements common to a pair of meshes. In this way it will be found that all squares of self impedance terms vanish, leaving only products of the type shown above. Since such an expansion produces some of the terms of the expansion of the determinant, the complete expansion of the general determinant Δ is linear with respect to all self impedances.

Coming to the numerator of any mesh current, as Δ_{I_k}, the same type of expansion can be applied likewise, using minors of four elements containing all the impedances common to any two meshes. The same result is obtained as before, that is, Δ_{I_k}, is linear with respect to all self impedances. One additional characteristic should be noted in regard to the numerator Δ_{I_k}, which will be apparent when the matrix of Δ_{I_k} is written out. This is the general determinant Δ with the kth column of A's replaced by the column of E's

$$\Delta_{I_k} = \begin{vmatrix} A_{11} & A_{12} & \cdots & A_{1,k-1} & E_1 & A_{1,k+1} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2,k-1} & E_2 & A_{2,k+1} & \cdots & A_{2n} \\ A_{31} & A_{32} & \cdots & A_{3,k-1} & E_3 & A_{3,k+1} & \cdots & A_{3n} \\ A_{41} & A_{42} & \cdots & A_{4,k-1} & E_4 & A_{4,k+1} & \cdots & A_{4n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ A_{n1} & A_{n2} & \cdots & A_{n,k-1} & E_n & A_{n,k+1} & \cdots & A_{nn} \end{vmatrix}.$$

The substitution of E's for the A's in the kth column has removed $A_{1k}, A_{2k} \cdots A_{kk} \cdots A_{nk}$. The elimination of A_{kk} means that Δ_{I_k} contains no impedances which are to be found in mesh k only. Thus the numerator of any mesh current is free of all impedances in that mesh, which do not couple with other meshes. However, in the general case, the current through an impedance is the summation of mesh currents which flow through that impedance.

It is then necessary to examine the form of the numerator of the resultant current which flows through a self impedance common to two meshes, for example k and p. To examine the terms involving these common impedances, the two minors including these impedances and the voltages are written out.

The common terms of Δ_{I_k} are engendered by

$$\begin{vmatrix} E_k & A_{kp} \\ E_p & A_{pp} \end{vmatrix} C(-1)^{k+p+k+p},$$

and the common terms of Δ_{I_p} are produced by

$$\begin{vmatrix} A_{kk} & E_k \\ A_{pk} & E_p \end{vmatrix} C(-1)^{k+p+k+p}.$$

The algebraic complements in both cases are identical since they were formed in both cases by the elimination of the kth and pth rows and columns of Δ.

If the mesh currents flow through the common self impedance in the same direction, the terms of the resultant current coming from the above minors will be

$$[E_k(A_{pp} - A_{pk}) + E_p(A_{kk} - A_{kp})]C(-1)^{k+p+k+p}.$$

The terms $(A_{pp} - A_{pk})$ and $(A_{kk} - A_{kp})$ produce cancellation of all self impedances common to meshes p and k. If the mesh currents flow through the common impedance in opposite directions, it is necessary to take their difference instead of their sum. In this case, however, half the common self impedances will differ in sign giving the same result. For instance, substituting the impedance terms of the network in Fig. 1, in the above minors, gives

$$\begin{vmatrix} E_k & -(Z_3 + Z_{13} + Z_{14} + Z_{23} + Z_{24} + Z_{34}) \\ E_p & Z_3 + Z_4 + 2Z_{34} \end{vmatrix} \times C(-1)^{k+p+k+p},$$

and

$$\begin{vmatrix} Z_1 + Z_2 + Z_3 + 2Z_{12} + 2Z_{23} + 2Z_{31} & E_k \\ -(Z_3 + Z_{13} + Z_{14} + Z_{23} + Z_{24} + Z_{34}) & E_p \end{vmatrix} \times C(-1)^{k+p+k+p}.$$

The numerator $\Delta_{I_{Z_3}}$ of the current flowing through self impedance Z_3, common to meshes k and p, is

$$\Delta_{I_{Z_3}} = \Delta_{I_k} - \Delta_{I_p}.$$

Accordingly, the terms in $\Delta_{I_{Z_3}}$ coming from the minors just written would be

$$E_k(Z_4 + Z_{34} - Z_{13} - Z_{14} - Z_{23} - Z_{24}) + E_p(Z_{14} + Z_{24} + Z_{34} - Z_1 - Z_2 - 2Z_{12} - Z_{23} - Z_{31}).$$

Thus the numerator $\Delta_{I_{Z_3}}$ of the current flowing through the common self impedance Z_3 is free of Z_3. Since the mesh currents can always be set up so as not to have more than two mesh currents flowing through any one impedance, the proof is sufficiently general. Therefore the numerator of any current is not only linear with respect to all self impedances in the circuit but also is free of all self impedances through which it flows.

Let all the impedances in the system remain fixed, but one self impedance, say Z_v. If the resistance component of Z_v remains constant and the reactive component varies, Z_v may be written as

$$Z_v = R + j\rho.$$

If the resistance component varies and the reactive component remains constant

$$Z_v = \rho + jX.$$

If the impedance varies at a constant phase angle

$$Z_v = Z_v'\rho,$$

in which Z_v' is a unit vector having the constant phase angle of Z_v. In all these cases ρ is a scalar variable, ranging from zero to plus infinity. The conditions above are the three possible cases of Z_v varying along a straight line in the complex plane. By segregating the terms free of ρ and factoring ρ out of the remaining terms, the general determinant Δ may then be written as

$$\Delta = \gamma + \delta\rho.$$

Similarly, if it is desired to obtain the current flowing through Z_3, for example, Δ_{I_k} and Δ_{I_p} are expanded into a constant term plus a constant term multiplied by ρ, that is,

$$\Delta_{I_k} - \Delta_{I_p} = \alpha + \beta\rho.$$

Then the current I_{Z_3} through Z_3 is

$$I_{Z_3} = \frac{\Delta_{I_k}}{\Delta} - \frac{\Delta_{I_p}}{\Delta} = \frac{\alpha + \beta\rho}{\gamma + \delta\rho}.$$

Thus I_{Z_3} follows a circular locus as ρ is varied. Therefore any current in the system follows a circular locus as any one self impedance is varied.

The voltage across the terminals of Z_3 is equal to the self impedance drop in Z_3 plus all other electromotive forces induced in it by coupling with other impedances. Representing this voltage by V_{Z_3} we have

$$V_{Z_3} = I_{Z_3}Z_3 + I_{Z_1}Z_{31} + I_{Z_2}Z_{32} + I_{Z_4}Z_{34} + \cdots I_{Z_n}Z_{3n}.$$

The current I_{Z_3} is free of Z_3 in the numerator. The remaining currents I_{Z_1}, I_{Z_2}, etc., are multiplied by mutual impedances only. Therefore the multiplication of the currents by impedances in this equation does not destroy the linearity of either numerator or denominator with respect to Z_3 or any other self impedance in the circuit. The voltage V_{Z_3} therefore follows a circular locus as ρ is varied.

Thus the summation of all self and mutual impedance drops between any two points of the system, producing the voltage between these two points, is a linear fractional transformation with respect to any one self impedance. If a source of electromotive force occurs between the two points across which the voltage is desired, it merely involves the addition of the constant electromotive force term to the summation of impedance drops, i.e. translation of the circle by the electromotive force vector.

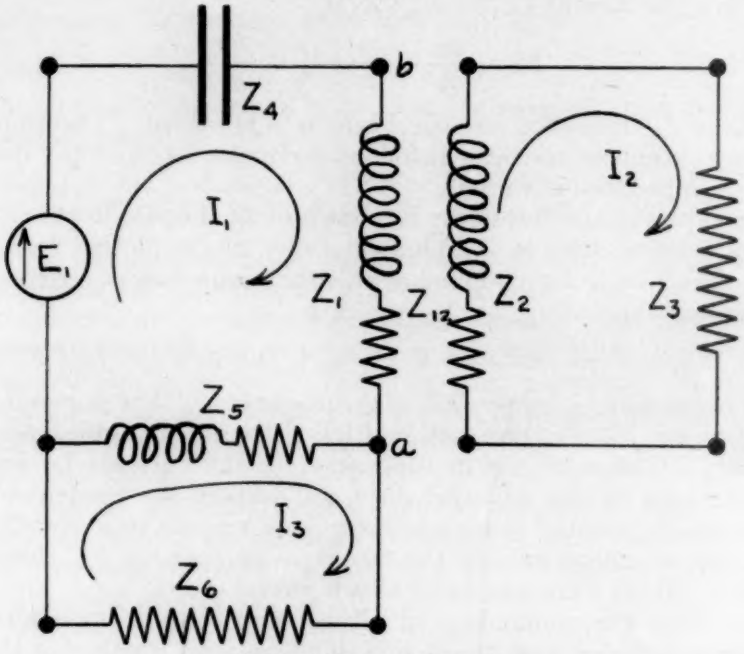
The result of this analysis may be summarized by the following statement:

In a network consisting of any number of linear and bilateral self and mutual impedance elements connected in any manner, with constant sinusoidal electromotive forces of like frequency connected in any arms, all currents and voltages

existing in the network follow circular loci when any one self impedance is varied along a straight line in the complex plane.

Example: The circuit drawn in Fig. 2 will be employed to

FIG. 2.



Circuit used in numerical example.

illustrate specifically how the self and mutual impedances enter into the solution of currents and voltages. The circuit contains one source of electromotive force E_1, self impedances Z_1, Z_2, Z_3, Z_4, Z_5, Z_6 of which Z_5 is common to mesh 1 and mesh 3 and a mutual impedance Z_{12} coupling meshes 1 and 2 as shown. The loci of the current through Z_5 and the voltage across Z_1 will be determined.

Summing up the potential differences in each mesh the following three equations are obtained:

$$\begin{aligned} I_1(Z_1 + Z_4 + Z_5) \quad &+ I_2 Z_{12} - I_3 Z_5 = E_1, \\ I_1 Z_{12} \quad &+ I_2(Z_2 + Z_3) = 0, \\ -I_1 Z_5 \quad &+ I_3(Z_5 + Z_6) = 0. \end{aligned}$$

Evaluating the general determinant

$$\Delta = \begin{vmatrix} Z_1 + Z_4 + Z_5 & Z_{12} & -Z_5 \\ Z_{12} & Z_2 + Z_3 & 0 \\ -Z_5 & 0 & Z_5 + Z_6 \end{vmatrix}$$

$$= (Z_2 + Z_3)[Z_5Z_6 + (Z_1 + Z_4)(Z_5 + Z_6)] - Z_{12}^2(Z_5 + Z_6).$$

The expression is linear with respect to any self impedance and contains the square of the mutual impedance term Z_{12}.

The specific determinants for I_1, I_2 and I_3 are

$$\Delta_{I_1} = E_1(Z_2 + Z_3)(Z_5 + Z_6) \qquad \text{(free of } Z_4\text{)},$$
$$\Delta_{I_2} = -E_1Z_{12}(Z_5 + Z_6) \qquad \text{(free of } Z_3\text{)},$$
$$\Delta_{I_3} = E_1Z_5(Z_2 + Z_3) \qquad \text{(free of } Z_6\text{)}.$$

To obtain the current I_{Z_5} flowing through Z_5 the currents I_1 and I_3 are added, giving

$$(1) \qquad I_{Z_5} = \frac{\Delta_{I_1} - \Delta_{I_2}}{\Delta} = \frac{E_1Z_6(Z_2 + Z_3)}{(Z_2 + Z_3)[Z_5Z_6 + (Z_5 + Z_6)(Z_1 + Z_4)] - Z_{12}^2(Z_5 + Z_6)}.$$

The self impedance Z_5 common to meshes 1 and 3 cancels out of the numerator of the current passing through Z_5.

The voltage across Z_1 is

$$(2) \qquad V_{ab} = I_1Z_1 + I_2Z_{12} = \frac{E_1(Z_5 + Z_6)[(Z_2 + Z_3)Z_1 - Z_{12}^2]}{(Z_2 + Z_3)[Z_5Z_6 + (Z_5 + Z_6)(Z_1 + Z_4)] - Z_{12}^2(Z_5 + Z_6)}.$$

Let the condenser in arm Z_4 be considered the variable self impedance and assume the following values for the remainder of the circuit.

$Z_1 = 5 + j50$ ohms $\qquad Z_4 = 0 - jX$ ohms
$Z_2 = 5 + j40$ " $\qquad Z_5 = 3 + j40$ "
$Z_{12} = 0 + j10$ " $\qquad Z_6 = 10 + j0$ "
$Z_3 = 10 + j0$ " $\qquad E_1 = 100 + j0$ volts.

Substituting these values in equation (1) the current

through self impedance Z_5 is given by

$$(3) \quad I_{Z_5} = \frac{15 + j40}{-77.275 - j53.450 + X(1.120 + j1.405)}.$$

Using equation (2) the voltage V_{ab} across Z_1 is

$$(4) \quad V_{ab} = \frac{6{,}172.5 + j6{,}065}{77.275 + j53.450 - X(1.120 + j1.405)}.$$

To plot these two circles the two invariant points ($X = 0$ and $X = \infty$) are used and some convenient third point, such as $X = 10$ ohms. Substituting these values of X in equations (3) and (4), the currents and voltages are as follows:

X ohms	I_{Z_5} amperes	V_{ab} volts
0	$0.455/-145.2°$	$92.1/9.8°$
10	$0.555/-141.4°$	$112.5/13.7°$
∞	0	0

The current and voltage circles are plotted in Figs. 3 and 4

FIG. 3.

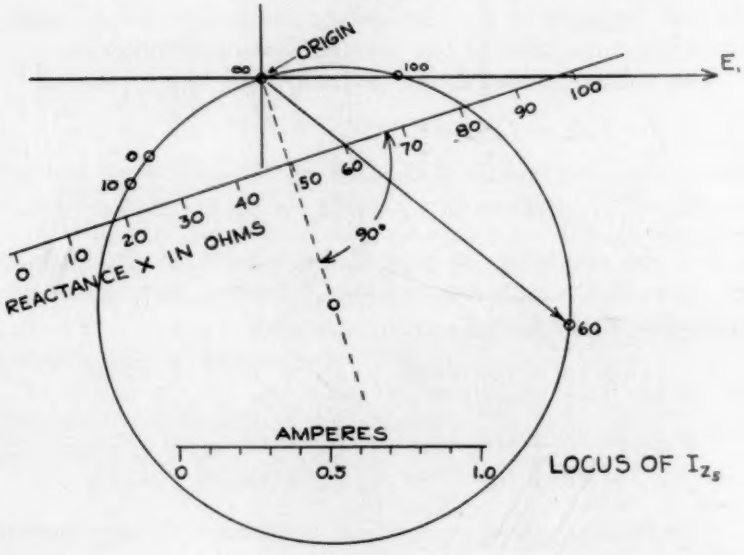
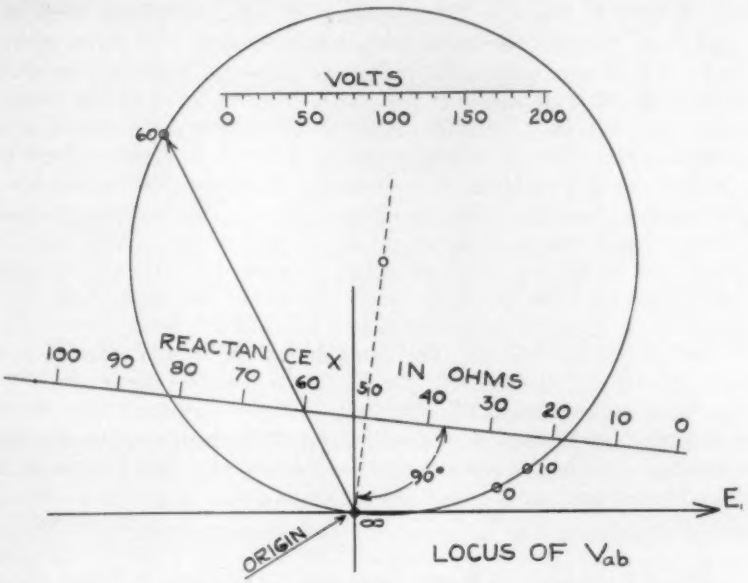


FIG. 4.



respectively. The intersection of the connecting lines between points on the circle and the point at infinity, for $X = 0$ and $X = 10$ ohms, on the scale line, fixes the corresponding interval of reactance in ohms on the scale line.

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Sun Heat Greatest on Mountains.—(*Heating and Ventilating*, Vol. XXXIII, No. 5.) Observations of solar radiation from the summit of Mount Aunconquilcha, Chile, nearly four miles above sea level, and where the solar radiation intensity in nearly one-sixth again as great as at sea level, have just been reported to the Smithsonian Institution. Delicate radiation-measuring instruments were carried to this crest at nearly 20,000 feet by C. P. Butler where he obtained readings intermediate between those obtained at sea level and those secured on loftiest balloon flights. The instrument used in the measurements was set up on a cake of ice. Although, of course, the result was to be expected, it seemed a little incongruous to be suffering from the cold and at the same time measuring solar intensity far greater than that which would have been recorded in the hot desert far below. The intensity of the solar radiation rose from 331 B.t.u. per sq. ft. per hr. at sea level—determined by numerous measurements in the past—to approximately 376 B.t.u. at the top of Mount Aunconquilcha. This tends to verify the estimates of changing solar radiation intensity in direct relation to altitude, deduced from other Smithsonian data.

R. H. O.

Diesel Locomotive Factory Goes Into Operation.—(*Power Plant Engineering*, Vol. XL, No. 5.) The first plant in the world for the exclusive manufacture of Diesel locomotives is now in operation at LaGrange, Ill. The Electro-Motive Corp. occupies an area of 74 acres. The plant was built to facilitate the application of progressive production methods in the manufacture of locomotives weighing 200 tons and its giant welded electric traveling crane is the largest ever operated on a welded structure. Electrically welded structural steel throughout the nine buildings included in the project affords an unusual application of welding for heavy structural work. The plant was built by the Austin Co. and much of the welding was done in their shop in Cleveland. From here, 104 ft. roof trusses and 65 ft. columns weighing 7 to 8 tons respectively, were shipped in prefabricated state. The plant is complete in every detail and with its own complete water supply system, modern sewage disposal plant, fire protection apparatus, source of industrial power supply and efficient heating plant, it could operate entirely independent of all public service facilities.

R. H. O.

OBJECT AND OBSERVER.

BY

A. A. MERRILL.

Consider this simple little sentence. We repeatedly say "I see the object," but how many of us know what we are talking about when we say that? What is this "I" which is the subject of the sentence? Is it our brain? If so, then the sentence mis-states the truth because the only thing the brain can note is an *effect* on some nerve sense terminal in the body holding the brain. Few of us realize that each human brain lives in a closed system with its own unique local standards (relativity) which is just why it is so hard for humans to understand each other.

Einstein has shown that in a sense, this is also true for physics, in that physicists in one gravitational field can note only *effects* on the senses of that field, which senses are scientific instruments. Einstein's work shows us how we must correct our local earth *percepts* if our astronomical predictions are to check accurately. Humans wear glasses for a similar reason. In our social life our error as to the meaning of this phenomenon is not an unimportant quibble, because this error is the cause of human conceit, intolerance and pre-event-dogmatism. While we may be justified in being dogmatic about the qualities of our own sense impressions (earth's instrumental records for physics) we are never justified in being dogmatic about the qualities of the object that produced the impressions. Let us watch our step. A large part of the friction between humans is due to the fact that we mix up the qualities of *effects* at our (the observer's) end of a space gap, with the qualities of an *object* at the other end of the same space gap. People often fight over different things, but think they are fighting over the same thing. One thing can not be equal to and different from itself at the same moment, but two *effects* produced by that thing may very well be different. Think of religious persecution due to

different interpretations of the meaning in terms of human action, of the same Bible. What an enormous amount of suffering humans have caused each other in pre-Einstein days, because they did not understand relativity. My own feeling is that the work of Einstein has started a new epoch in the intellectual and social life of mankind.

There are always two ends to the space gap between *object* and *observer;* do not mix them up and treat them as if they were the same end. Our knowledge is tied hard and fast to *effects* at our (the observer's) end; do not project outwardly, qualities of *effects* at the observer's end and think they are qualities of the object. Unfortunately this has been done by a few scientists writing on relativity and it is done continually by laymen in our social life. In our social life it causes an enormous amount of friction, the upper limit being war. Einstein has shown us what trouble this blunder causes for physics, but few of us realize that this same blunder of mixing *effects on an observer* with the *object* causing the effects, produces continual squabbling among humans.

MEDAL DAY MEETING.

The annual Medal Day Meeting of The Franklin Institute was held in the hall of the Institute at 3:30 o'clock on the afternoon of May 20, 1936. The meeting, which was also the regular monthly meeting, was called to order by the President, Mr. Nathan Hayward.

The President: In the annals of The Franklin Institute, the exercises this afternoon are known as Medal Meeting. This day is set apart as the time at which are made the awards recommended during the year by our Committee on Science and the Arts.

This Committee has a long and enviable record. Its mistakes have been few and its wisdom great. Since 1919 the Secretary of this Committee has been Dr. George A. Hoadley to whom full credit should be given for his guidance of the workings of this Committee.

On Monday of this week, Dr. Hoadley, at the age of 87, —still in harness, still serving the Institute—died.

As a tribute of respect to his memory, I would ask the audience to rise.

The President then announced that all awards recommended by the Committee on Science and the Arts during the current Institute year would be made at this time.

Award of Longstreth Medals.

To Prof. Alfred V. deForest, President, Magnaflux Corporation, New York City, and Major William E. Hoke, Consulting Engineer, Baltimore, Maryland. Mr. Clifford W. Bates, sponsor.

The President called upon Mr. Bates.

Mr. Bates: "Mr. President, it is my privilege to sponsor Professor V. deForest, President of the Magnaflux Corporation of New York City, and Major William E. Hoke, Consulting Engineer of Baltimore, Maryland, who have been voted Longstreth Medals, 'in consideration of the new application of certain long-known principles to fill the need for a

ready means of detection of hidden defects, primarily at or near the surface of magnetic materials and of the development of means for the commercial application of this method to present day engineering problems.'"

The President presented the medals as follows: "Major Hoke, Professor deForest, by power vested in me as President of The Franklin Institute, I have the honor to present to you the Edward Longstreth Medal."

Professor deForest and Major Hoke expressed their appreciation.

To Mr. Peter P-G. Hall, President, The Hall Planetary Company, Philadelphia, Pa. Mr. W. P. Valentine, sponsor.

The President called upon Mr. Valentine.

Mr. Valentine: "Mr. President, the Committee on Science and the Arts has recommended Mr. Peter P-G. Hall, President of the Hall Planetary Company of Philadelphia, for the Longstreth Medal, 'in consideration of his invention and development of machine and cutters for planetary milling and threading.'"

The President presented the medal as follows: "Mr. Hall, in accordance with the citation just read, I am pleased to present to you our Longstreth Medal. You have the distinction of being the only Philadelphian honored by the Institute this year."

Mr. Hall expressed his appreciation for the honor conferred upon him.

To Mr. Elmer A. Sperry, Jr., Vice-President, Sperry Products, Inc., Brooklyn, New York. Mr. W. Laurence LePage, sponsor.

The President called upon Mr. LePage.

Mr. LePage: "Sir, the Board of Managers of The Franklin Institute has sanctioned the award of a Longstreth Medal to Mr. Elmer A. Sperry, Jr., Vice-President of the Sperry Products, Inc., Brooklyn, New York, 'in consideration of his initiative and for his work in the development of blind flying instruments employing gyroscopic principles, which development includes the gyroscopic horizon, directional gyro, and automatic airplane pilot.'"

The President presented the medal as follows: "It gives me especial pleasure to honor the son of an honored father.

It was my agreeable task to present a Cresson Medal to your father in nineteen hundred and twenty-nine. Apparently the gyroscope has stabilized the family!!"

Mr. Sperry responded: "Mr. President, Members of The Franklin Institute, it is with a great deal of pleasure that I stand here where my father stood to receive a medal, in fact two medals, from The Franklin Institute.

"I want to thank the Committee for its investigation and recommendation. But of course I know that the real people who should receive the Medal are my associates, who 'carry on' for us as they did so wonderfully for my late father."

AWARD OF WETHERILL MEDAL.

To Mr. Albert L. Marsh, President, Hoskins Manufacturing Company, Detroit, Michigan. Mr. Edward L. Forstall, sponsor.

The President called upon Mr. Forstall.

Mr. Forstall: "Mr. President, Mr. Albert L. Marsh, President of the Hoskins Manufacturing Company, Detroit, Michigan, is recommended to you for the award of the Wetherill Medal, 'in consideration of the contribution of a material which has proved of extreme importance to the electrical industries.'"

The President presented the medal as follows: "Mr. Marsh, in presenting to you the John Price Wetherill Medal, we are glad to pay grateful recognition to your persistent efforts which have resulted in the development of 'chromel,' without which our electrical appliances, domestic and industrial, could hardly have been perfected as they are today."

Mr. Marsh expressed his thanks.

AWARD OF THE CLARK MEDAL.

To Dr. Joseph Becker, President, The Koppers Construction Company, Pittsburgh, Pa. Mr. Owen B. Evans, sponsor.

The President called upon Mr. Evans.

Mr. Evans: "Mr. President, it is my privilege to present to you Dr. Joseph Becker of the Koppers Construction Company of Pittsburgh, Pennsylvania, as the recipient of the Clark Medal, 'in consideration of his improvements in

the art of the carbonization of coal and manufacture of gas in coke ovens, and particularly for his work in the development of the oven known as the Becker Oven.' "

The President presented the medal as follows: "Dr. Becker, I always have great pride in presenting the Walton Clark Medal, for it was founded in honor of another outstanding engineer in the gas industry, a man who was near to us all. I am happy, Sir, to present it to you for your important achievements."

Dr. Becker expressed his deep appreciation for the honor conferred.

AWARD OF LEVY MEDAL.

To Dr. Mayo D. Hersey, Brown University, Providence, Rhode Island. Dr. Thomas D. Cope, sponsor.

The President called upon Dr. Cope.

Dr. Cope: "Mr. President, I sponsor the award of the Levy Medal to Professor Mayo D. Hersey of Brown University, Rhode Island, 'for his series of papers on "A Short Account of the Theory of Lubrication," published in the June, July, August, and September issues of the JOURNAL of the Institute for 1935.' This medal is presented by the Institute each year for that paper appearing during the preceding year in the JOURNAL OF THE FRANKLIN INSTITUTE, which is voted to be the most meritorious in point of research and literary style."

The President presented the medal as follows: "Mr. Hersey, members of The Franklin Institute always look with interest at the recipient of the Edward Levy Medal which I now present to you. Having read your paper in the JOURNAL, we are glad to pay this tribute to the author."

Mr. Hersey responded: "I greatly appreciate the honor. Through a curious coincidence I have just received a first page proof of the papers which were published in the JOURNAL and which are now being made into a little booklet. I shall be delighted to present a copy of that booklet to the Library of The Franklin Institute to express my appreciation."

AWARD OF THE POTTS MEDAL.

To Dr. Felix A. Vening Meinesz, University of Utrecht, Utrecht, Holland. Dr. Charles B. Bazzoni, sponsor.

The President called upon Dr. Bazzoni.

Dr. Bazzoni: "Mr. President, upon the recommendation of the Committee on Science and the Arts the Potts Medal this year has been voted to Dr. Felix A. Vening Meinesz, of the University of Utrecht, Holland, 'in consideration of the outstanding value of his work in geodesy; and the energy, originality, and skill he has shown in the development and use of apparatus for determining gravity at sea.' Unfortunately, Dr. Vening Meinesz was unable to make the trip to America to receive his medal in person. He is therefore represented by Dr. P. J. Groenendaal, Consul for the Netherlands in Philadelphia. Dr. Groenendaal."

The President presented the medal as follows: "Mr. Consul, in the name of The Franklin Institute I am delighted to present to you the Howard N. Potts Medal, for transmission to your distinguished countryman. We appreciate your kindness in receiving it in his absence."

Dr. Groenendaal responded: "Mr. President, I feel highly honored in being here to represent my countryman to whom you have awarded this Medal. I believe he is the only foreigner to receive such distinction today. I accept the Medal in his behalf and shall transmit it to him without delay."

The President then called upon the Secretary to read a message which had been received from Dr. Vening Meinesz.

Message from Dr. Felix A. Vening Meinesz, Recipient of the Potts Medal.

"I wish to express my sincere thanks for the great honor, which The Franklin Institute of the State of Pennsylvania is willing to confer on me by awarding to me the Howard N. Potts Medal.

"My scientific career and especially my main life-work, the determination of gravity at sea, has brought me already into intimate contact with many of the most distinguished scientists and men of your country. I wish to mention in the first place the name of Dr. William Bowie, Chief Division of Geodesy of the Coast and Geodetic Survey and President of the International Union for Geodesy and Geophysics, to whom I am greatly indebted for much interest shown in my work, for

much support, leading for instance to several expeditions with U. S. Submarines for this purpose, and for much friendship. Further the names of Dr. Fred Wright, my good friend and companion of the expedition of the U. S. Submarine S 21, Dr. Richard M. Field and Dr. Harry Hess, both met and befriended with during the expedition of the U. S. Submarine S 48, Mr. Walter D. Lambert, Mr. Clarence H. Swick and Mr. N. H. Heck of the U. S. Coast and Geodetic Survey, Dr. Arthur L. Day, Director of the Geophysical Laboratory of the Carnegie Institution, Admiral Freeman and Capt. Hellweg, formerly Heads of the Naval Observatory, Admiral Gherardi, formerly Head of the Hydrographic Office and many good friends in the submarine Service of the U. S. Navy. And I am omitting many names I should like to mention! I am feeling deeply indebted to the U. S. Navy for organizing the submarine expeditions that I have already mentioned. In 1925 the Washington Academy of Sciences has been kind enough to nominate me honorary member.

"So my scientific career has brought me much support and many friends in the United States. Three times I have stayed in your country and I have always been so kindly received that I felt myself at once at home and that I deeply regretted to leave. I wish to send a message of sympathy to your great country, to the building of which I am proud to realize that my own small country has been able to contribute, and a message of sincere thanks to your Institute for this new proof of appreciation and goodwill."

AWARD OF THE CRESSON MEDALS.

To Dr. George O. Curme, Jr., Vice President, Carbide and Carbon Chemicals Corporation, New York City, New York. Mr. W. H. Fulweiler, sponsor.

The President called upon Mr. Fulweiler.

Mr. Fulweiler: "Mr. President, the Board of Managers has sanctioned the award of the Cresson Medal to Dr. George O. Curme, Vice-President, and Director of Research, of the Carbide and Carbon Chemical Corporation, New York City, 'in consideration of his foundation of a new technology which has resulted in the development of a rapidly expanding industry; namely, that of synthetic aliphatic compounds based

upon the olefines as starting material; and his development of profitable uses for these compounds in industry.'"

The President presented the medal as follows: "Dr. Curme, upon recommendation of our Committee on Science and the Arts, I have the honor to present to you the Elliott Cresson Gold Medal, in recognition of your untiring efforts in research and your able guidance in development in the field of chemistry, which have given to our country leadership in certain branches of this very important science."

Dr. Curme responded: "I thank you for the recognition. I appreciate the personal honor involved but I wish at this time to give public recognition to my staff, without whom this work would not have been possible."

To Dr. Robert J. Van de Graaff, Massachusetts Institute of Technology, Cambridge, Massachusetts. Mr. Walter C. Wagner, sponsor.

The President called upon Mr. Wagner.

Mr. Wagner: "Sir, the Board of Managers has also approved the award of the Cresson Medal to Dr. R. J. Van de Graaff, Associate Professor of the Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts, 'in consideration of his development of an electrostatic generator for the production of high voltage direct currents, through which he has made possible the extension of nuclear investigation.' In his absence we ask that you hand the Medal to the Secretary of the Institute to be forwarded to the recipient."

Mr. Hayward: "Another Cresson Medal has been voted by our Committee, as you have just heard. This one is awarded to a young gentleman who has presented a worthy excuse. We are very pleased, none the less, to have this opportunity to show our appreciation of the splendid work being done in the Massachusetts Institute of Technology. Mr. Secretary, the Medal. . . ."

Mr. Allen: "I shall forward it, Mr. President, with proper ceremony."

AWARD OF FRANKLIN MEDALS.

To Frank Baldwin Jewett, Ph.D., D.Sc., D.Eng., LL.D., President and Director, Bell Telephone Laboratories, Inc.,

New York City, New York. Dr. Frederic Palmer, Jr., sponsor.

The President called upon Dr. Palmer.

Dr. Palmer: "Mr. President: I have the honor to present to you, upon the recommendation of the Committee on Science and the Arts, Doctor Frank Baldwin Jewett, Vice President of the American Telephone and Telegraph Company, President and Director of Bell Telephone Laboratories, Incorporated, as the recipient of a Franklin Medal 'in recognition of his many important contributions to the art of telephony, which have made conversation possible not only from coast to coast, but from this country to the other side of the world,—contributions of which some were made by him alone, and some through the collaboration of other workers in the great laboratory of research which he organized and which he has directed with such signal success.'

"When Dr. Jewett entered the employ of the American Telephone and Telegraph Company in 1904, he brought to the telephone industry a mind thoroughly trained in scientific procedure and a contagious enthusiasm for surmounting difficulties. His qualifications and ingenuity were soon put to the test. At that time the longest distance from which speech was possible was from New York to Chicago—950 miles. It was the problem of the company to discover means whereby New York and San Francisco, a distance of *3000 miles*, could be linked by such communication. A research laboratory was organized to aid in the solution of the many problems involved in such an extension. In two years the distance was doubled, and service was in operation between New York and Denver. Still it was not known how to span the remaining 1000 miles. Nevertheless, in January, 1915, there was established successful service between New York and San Francisco.

"This phenomenal extension of commercial telephone service in such a short time was accomplished, largely under Dr. Jewett's direction, by the introduction of phantom loading and the loading of large-gauge and open-wire circuits, by the use of telephone amplifiers on loaded lines, and by the development of phantom duplex cables.

"Before the end of 1915 the first words were spoken from

Arlington to Paris, though it was twelve years more before there existed a foreign commercial telephone service.

"During the next ten years came many of the most important advances in the field of communications, including the development of the vacuum tube, improvements in the art of inductive loading, building of transcontinental telephone lines, development of the telephone repeater, introduction of machine switching on a large scale, and development of high speed submarine telegraph cable. While credit for such advances as these should be given whole-heartedly to many of the workers in the research laboratories, nevertheless it is true that as an engineer Dr. Jewett had a large personal share in all of these developments, and as an executive helped to weld diversified units of the Bell System into a well balanced whole, thereby achieving for the telephone system of this country a position of world leadership.

"Fortunately for the advance of civilization, the scientific destinies of the American Telephone and Telegraph Company for the last quarter of a century were largely in the hands of the late General J. J. Carty, with whom Dr. Jewett was closely associated. Dr. Carty was distinguished for his remarkable clarity of vision, unusual ability in organization, and tenacious will to succeed. Twenty years ago the Franklin Medal was awarded to him. Dr. Jewett is outstanding for the same characteristics which characterized his associate. Today we are happy to recommend the award of a Franklin Medal to Dr. Jewett."

The President presented the medal as follows: "By authority vested in me as President of The Franklin Institute, I have the honor and great personal pleasure to present to you the highest award the Institute bestows, and with it a Certificate of Honorary Membership. Because your genius has touched every one of us, it is particularly gratifying to pay homage to you, for your vision, your patience, your endless energy, which have drawn the world together and have made it possible for the humblest in the land to hear the utterances of the mightiest."

Dr. Jewett: "In accepting this medal I will delay until later in the proceedings an expression of my appreciation."

To Charles Franklin Kettering, D.Eng., D.Sc., Director,

General Motors Research Laboratories, General Motors Corporation, Detroit, Michigan. Dr. Frederic Palmer, Jr., sponsor.

The President called upon Mr. Palmer.

Mr. Palmer: "Mr. President: Your Committee on Science and the Arts recommends that a Franklin Medal be awarded to Charles Franklin Kettering, Vice President of the General Motors Corporation, General Director of General Motors Research Laboratories, Vice President of the Frigidaire Corporation, 'in recognition of his significant and timely contributions to the science of automotive engineering,—a science out of which has grown the greatest industry in this country, the manufactured product of which has, in only a quarter of a century, changed the face of the civilized world.'

"To the field of transportation engineering, Mr. Kettering has contributed the electric self-starter for automobiles, ethyl gasoline, the chief features of the design of the Liberty motor, and the adaptation of the crude-oil-burning Diesel motor for transportation both by land and water. Through his organization and direction of what is now known as The Research Section of the General Motors Corporation many other contributions have been made to the field of automotive engineering.

"In addition to these, we owe to Mr. Kettering's inventive genius the automatic cash register, the Delco lighting system and other electric power adaptations for rural communities, as well as Frigidaire.

"Taking these inventions all together it may be stated that a large fraction of the inhabitants of this country are indebted in one way or another to Mr. Kettering for providing them with what they regard either as daily necessities or as luxuries or both.

"Mr. President, it is a great pleasure to present to you a man whose modest personality has made him beloved by all those who work under his direction, and whose remarkable versatility in invention makes him beloved by millions of those to whom he is only a name—Charles Franklin Kettering."

The President presented the award as follows: "Dr. Kettering, it is my privilege as President of The Franklin Institute to present to you, Sir, the Franklin Medal, together

with a Certificate of Honorary Membership in this old Society. It is most fitting that we who are trying to make known to the layman the principles of science underlying everyday life, should pay highest tribute to you who have devoted your life to the development of an industry which, in large part because of your brilliant inventions, has changed the whole set-up of our present-day civilization.

Dr. Kettering: "I, too, will delay formal expression of appreciation."

After the presentation of the medals two papers were read having been prepared by the Franklin medalists of the day.[1]

After an expression of appreciation to the medalists and the audience for their attendance, the President declared the meeting adjourned.

MEDAL DAY DINNER.

Wednesday Evening, May 20, 1936.

For the first time in the history of the Institute, Medal Day Dinner was held this year in its own building. Franklin Hall, dignified and impressive, made a beautiful setting for such an event. At one end, seated at a long table, were all of the medalists who had been honored in the afternoon. Mr. Nathan Hayward, President of the Institute was Toastmaster.

Mr. Hayward spoke as follows:

"As we open this dinner, there is one thought in the minds of all—Howard McClenahan.

"It was his thought to have Medal Day once a year, followed by a dinner. And all the dignity which these occasions may bear, and all the atmosphere that has grown around them, is due to him. The personal loss which I have had I cannot dwell on at this time and place, but I must mention that at these dinners, or at any of these functions, when it seemed wise to get a little lightness into the proceedings, I could always rely on the knowledge that a few sly digs at him would be understood and well taken, and returned with equal good nature but better wit!

"So much of our present activities were started by Mc-

[1] See pages 127 and 133, this issue.

Clenahan during his ten years of association with The Franklin Institute. The museum was his life and he gave his life for it. Without his courage and imagination it could never have existed.

"The Bartol Research Foundation was opened under his supervision, and its present success is but the outgrowth of the devoted efforts of Charles Day, W. C. L. Eglin, and Howard McClenahan, whose wisdom outlined the policies and chose Swann to run it.

"It was due to Howard McClenahan that the Biochemical Research Foundation joined our forces.

"He is no longer with us as a forceful personality, but his spirit lives on and will ever be a part of the Institute. We should all be inspired to continue his work, in making that spirit live and grow.

"Medal Day in The Franklin Institute has come to be regarded as one of the outstanding scientific events in the United States. Year after year, the Committee on Science and the Arts faithfully considers cases of invention, of discovery, of scientific development, and then reports its findings to the Board of Managers, together with recommendations for awards. I may say here that the Board never disputes the findings of that Committee. It is made up of men who give unsparingly of their time and who make painstaking researches to safeguard their reports.

"The recipients of medals from the Institute have expressed the greatest gratification upon being so honored, and the Institute feels that the value of its medals are enhanced by the brilliant records of the recipients!

"As an innovation this year, we have all of our Medalists at the head table, in order that our members and friends may see and, we hope, concur in the wisdom of the Institute's choice:

Mr. deForest, Mr. Hoke, Mr. Hall, Mr. Sperry, all Longstreth Medalists
Mr. Marsh, our Wetherill Medalist
Dr. Becker, who received the Clark Medal
Mr. Hersey, whose paper you have read in the JOURNAL—Levy Medalist

Dr. Groenendaal, who represents Dr. Vening Meinesz, the Potts Medal, our only foreign award this year

Dr. Curme, who received the Cresson Medal, our oldest award

"Frequently the Committee recommends some foreign scientist as the person to receive the Franklin Medal but this year in its wisdom it has seen fit to keep the honors at home, with the result that we have the unusual pleasure of having with us two gentlemen who have in a very real and vital way influenced our daily lives. As I introduce these two recipients of our highest honor, I am going to ask them to address you.

"I think the large gathering we have here tonight speaks well for the popularity of our Medalists. For one gentleman I can speak from close personal contact and am delighted to say that, in my opinion, because of my knowledge of this one man, the mechanical wonders which come from our great industrial laboratories are dimmed in comparison with the brilliant personalities which create them. I am sure you will agree with me when I call upon one of our Franklin Medalists of the day—Dr. Jewett, Bell Telephone Laboratories."

Dr. Jewett, President of the Bell Telephone laboratories, gave a very interesting talk on modern scientific developments. He spoke especially about the marvels of transportation and referred to the inventions which have combined to give us our present-day automobile.

The Toastmaster then introduced the next speaker:

"The next gentleman it is my privilege to present to you is a very modest person, who prefers to do things in his laboratory rather than to talk about them in public. But just the same I am going to ask him if he won't tell us 'what makes the wheels go round.' Dr. Kettering, General Motors Company who today was also awarded the Franklin Medal."

Dr. Kettering gave an amusing talk on mechanics of our times and dwelt especially upon the inventions which have characterized the Bell Telephone Company in recent years. He expressed the hope that the men who have been able to extend transmission of speech halfway around the world would next work on some method of getting sound transmitted from the inner ear to the brain. Dr. Kettering said he knew of cases where it took years for a sound to penetrate. He made many other entertaining and pertinent remarks on present-day conditions.

Mr. Hayward: "As you entered our building this evening, you may have noticed the inscription above the main entrance, 'In Honor of Benjamin Franklin.' Thus we proclaim to the world that Franklin is our patron and we are proud to do him honor. Other organizations have been banded together for the same person."

"We had expected to have the president of such a society with us this evening. It is known as the 'International Benjamin Franklin Society,' and has its headquarters in New York City. It was founded by William Guggenheim to further the teachings of Franklin. Mr. Guggenheim has sent us the following greetings:

" 'Regret exceedingly I cannot be with you on your Medal Day celebration. Your purpose in creating thereby a Benjamin Franklin revival should meet with a hearty response throughout the nation as it should influence a return to Franklin's wholesome political, financial and economic views. The present grotesque situation is becoming serious and you have my hearty indorsement and best wishes for great success.'

"Unfortunately, Mr. Stout, President of the International Benjamin Franklin Society, was also compelled to be absent from our dinner because of illness. He has, however, sent us his paper which will be read by Mr. W. H. Fulweiler, a member of our Board of Managers."

Mr. Fulweiler then read an interesting paper in which Mr. Stout had pointed out that Benjamin Franklin in his scientific ideas was quite cognizant with present-day thoughts and activities. He gave many illustrations to prove his point. The demonstration was greatly enjoyed.

Mr. Hayward: "Ladies and Gentlemen, and particularly members of the Institute, the next speaker should claim your special interest, for he is the man who has been chosen by the Board of Managers to guide the workings of this old Society. Although he is well known to many of you, and has been for years a valued member of our organization and a regular attendant at Medal Day since its inception, tonight he appears in a new role—that of Secretary and Director of The Franklin Institute. Mr. Henry Butler Allen."

MR. ALLEN: "We have seen this afternoon in our Medal Day activities the results for the year of one of The Franklin Institute's activities.

"I will touch briefly on the other work during the year.

"In the scientific research field our Bartol Laboratories under Dr. Swann have made outstanding contributions to our knowledge of cosmic rays.

"The Biochemical Research Foundation under Dr. McDonald is in the midst of important findings about that border line between living organisms and chemistry.

"Our library has been opened evenings and is having steadily increasing attendance.

"Our lectures were well attended and contributed a wide range of subjects.

"Our new Museum or 'Wonderland of Science' and Planetarium attendance for the past twelve months has been 370,000 persons of which 100,000 were Philadelphia school children and 30,000 out of town school children. Neighboring colleges are extending their use of our facilities.

"I wish I had time to tell you about some of the special activities which have been going on and which seem to prove their usefulness, such as, our new Music Section with its lectures and record making, amateur telescope building, amateur aeronautics, cooperation with the Federal Bureau of Investigation in Science in Crime Detection, the special Christmas Planetarium, the Great Chalice of Antioch, etc.

"Our membership, through the splendid work of our Membership Committee under the enthusiastic lead of Mr. Wetherill, increased from 1,800 a year ago to 6,000 today.

"A continuing increase in public interest is being shown. This to my mind shows that we are right in trying to live up to Benjamin Franklin's expressed purpose in life, to be useful to his fellow men.

"We are now actively working toward a very important event. That will be the dedication of this Benjamin Franklin Memorial on Franklin's birthday in January 1938.

"The Poor Richard Club and the Institute are cooperating in planning this. This hall, which was the original idea of the Poor Richard Club is the center of the Memorial. Here will stand the heroic statue now being made by Fraser.

"The Memorial includes besides this, research activities, libraries, lectures, publications, medal awards, and a great science museum—all working to collect and disseminate knowledge for everybody to understand.

"Franklin was a distinguished scientist, but not in the limited sense of the then so-called natural sciences. He possessed the *scientific approach* to all problems of life; the logical analysis which leads to sound thinking.

"Franklin's great part in founding our country resulted from his self-acquired ability to *think straight.*

"A living vital Memorial dedicated to these ends will be, as Franklin might have phrased it, of much needed service."

MR. HAYWARD: "You will all recall that in the campaign for funds in 1930, two gentlemen were outstanding in their efforts to make that drive the great success that it was. Without their enthusiasm and devoted labors, we could not have gotten very far. One was Cyrus H. K. Curits, who crowned a life time of achievement by making this memorial a reality, and the other was Mr. George Wharton Pepper, who headed the campaign committee and did valiant service for our cause. Mr. Pepper, may we hear from you?"

MR. PEPPER: "I have no addition to make to what Mr. Allen has said so well. No addition, no subtraction—and no division. But by way of multiplication I should like to emphasize the announcement he has made.

"It sometimes happens that people live and move at a certain time without realizing that their age is one of the great eras of history. Sometimes one may even be an actor in a transaction destined to become historic without realizing that it is actually taking place. Likewise announcement may be made in our hearing by a man of modesty and reserve without full appreciation on our part of the significance of his utterance. I hope that when we leave this hall tonight it will be with a lively appreciation of what Mr. Allen's announcement means to Philadelphia and to the United States.

"These are days of divergent opinion and even of bitter discord. We know that national recollections are the foundation of national greatness. But when we summon the Great Dead to our aid, their very names in many cases awaken dissension and quicken disagreement. Washington and Hamilton, Jefferson and Madison, Marshall and Taney—these today are controversial names; and Lincoln is thought of at the moment merely as the founder of a political party. Almost alone among our great men stands Benjamin Franklin,

the peace-maker, the Apostle of Unity. As his influence was potent for unity in the Federal Convention of 1787 so we need a reincarnation of Franklin's spirit in our own day to serve as a unifying influence throughout the Republic. Mr. Allen's announcement means that this reincarnation is actually to be achieved; that Franklin will live again in our midst; that from this center his influence will radiate to every part of the nation; and that however much we differ about other men we are to be united in our loyalty to him.

"It is appropriate that the reincarnation of Franklin should occur in Philadelphia, not merely because it was here that he lived and wrought but because large plans for the quickening of the life of Philadelphia are being made by our Mayor who is manifesting a truer appreciation of the greatness of our City than we native-born Philadelphians sometimes show.

"And it is especially appropriate that this building should be the shrine of Franklin's reincarnation. It is the result of the cooperation of some 11,000 citizens of Philadelphia, who, at the suggestion of the Poor Richard Club and under the leadership of Mr. Cyrus H. K. Curtis, did tardy justice to Franklin's memory and created a living memorial to the man who did so much for his country and will do still more when Mr. Allen's plans are matured and put into full effect.

"Just think of it! We are not preparing funeral rites for a dead hero. We are not proposing to dedicate a monument to a dead Past. We are working in a living Present in the confident expectation of a fruitful Future. This is an enterprise in which we all can with enthusiasm enlist. Citizens of Philadelphia! Can we not once more assert the primacy which was ours in Colonial Days? Let us all pledge to Director Allen our cordial support—and let us show by our works that we are of one blood with those who made America!"

MR. HAYWARD: "On behalf of The Franklin Institute, I wish to thank Mr. Pepper for a splendid message. I hope we may all have the pleasure of gathering together on January 17, 1938, for the dedication of this building which typifies so much for the citizens of Philadelphia."

Bethlehem Introduces New Process for Electrolytic Coating of Steel Wire with Zinc.—(*Iron Age*, Vol. 137, No. 17.) The Bethlehem Steel Co. recently opened a new mill for the manufacture of Bethanized wire and fence at Johnstown, Pa. The raw materials used in the process consist of wasted zinc concentrate carrying 50 to 60 per cent. zinc and a long list of other elements which have a possible metallurgical or economic bearing on the operation of the process. An acid zinc sulphate solution is used and the peculiar phenomenon of overvoltage is employed in overcoming, in a sense, the laws of electro-chemistry in obtaining a perfect coating of pure zinc on steel. This overvoltage is a property that certain materials have of opposing the evolution of hydrogen at their surface. The hydrogen overvoltage of iron is normally below the potential of zinc on the hydrogen scale. It rises with increasing current density and by sufficiently increasing this factor zinc may be deposited. The unit at Johnstown has two plating cells, each 110 ft. long and taking 40,000 amp. at normal load. There are normally 12 wires in each cell travelling at speeds of from 50 to 200 ft. per min. The electrolysis is carried out at current densities from 700 to 2,000 amp. per sq. ft. The anodes consist of an alloy of lead and silver which is far more resistant then lead alone and accounts for the extremely small quantities of lead found in the finished product. Uncoated drawn wire first goes to fused caustic pots in which it is cleaned and annealed. The two pots provide a total length of about 40 ft. After washing, the wire goes to the plating cells. After plating the wire has a mat or finely crystalline surface. In order to expose as small as possible an area to corrosion the wire is rendered smooth and bright by polishing or drawing.

R. H. O.

50,000,000 Kw.-Hr. for Subway Lighting.—(*Transit Jl.*, Vol. 80, No. 5.) Enough power to light a city of 75,000 people is used every year by the Interborough Rapid Transit Company, New York, to illuminate its elevated platforms and subways. Altogether some 50,000,000 kw.-hr. are used annually to keep lamps burning in 340,000 sockets, at a cost of more than $500,000 a year. Every day 2,000 lamps burn out, making the company the fourth largest direct consumer of lamps in the country. The largest single use of lamps is on the subway and elevated platforms where there are approximately 153,000 sockets. In the 3800 subway and elevated cars there are 136,000 light outlets. Thousands of lamps are also used to insure safety of operation, indicators on telephone switchboards, etc.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

CONFERENCE OF AGRICULTURE, INDUSTRY, AND SCIENCE.

The Bureau was represented by Frederick Bates at the Second Conference of Agriculture, Industry, and Science May 12–14, inclusive, held under the sponsorship of the Farm Chemurgic Council and The Chemical Foundation, at Detroit, Michigan. Approximately a thousand delegates were in attendance. The session attracted nationwide attention, representing a cross-section of the scientific, technical and executive interests of the country drawn together with the common purpose of creating and furthering new developments for products of the soil.

The two major subjects, if such can be selected, were power alcohol and sugar developments. Among the outstanding addresses were those by Heber J. Grant, president; L. D. S. Church, Salt Lake City; and Wm. J. Cameron of the Ford Motor Company. Other important contributions were: The American Farm Problem, by Dr. Alonzo E. Taylor, director of the Food Research Institute, Stanford University; The Application of Physics to Agriculture, by Dr. George R. Harrison, professor of physics, Massachusetts Institute of Technology; What Agriculture Owes to Science, by Edward A. O'Neal, president of the American Farm Bureau Federation; Science in Industry, by C. M. A. Stine, vice-president, E. I. du Pont de Nemours and Company; The Relation of Power Alcohol to Our Economic Problems, by Francis P. Garvan, president of the Chemical Foundation; Jerusalem Artichokes, by Dr. W. L. Burlison, head of department of agronomy, University of Illinois.

The larger general session of the Council was given over to a very broad discussion of the subject of power alcohol. This included not only the question of the economic feasibility and advisability of the use of anhydrous ethyl alcohol in motor fuels, but also thoroughly covered the technique of

* Communicated by the Director.

alcohol production, including available and possible sources of raw materials. The conviction was apparent among the numerous experts present that the use of alcohol in American motor fuels is inevitable and imminent.

The subject of the sugars and the starches occupied one session, and practically all phases of America's diverse sugar and starch industries were represented. A thorough and constructive discussion on these subjects finally became concentrated on methods for increasing the production of sugar in the United States. A valuable paper on the general subject had previously been presented by Heber J. Grant of Salt Lake City. The Bureau was given permanent committee representation.

ORGANIC PLASTICS.

The modern plastics industry deals chiefly with moldable materials manufactured from organic compounds. Synthetic resins, natural resins, cellulose derivatives, and protein substances are the four principal types of these organic plastics. The inorganic molding materials, such as concretes, cements, and ceramics, and also rubber (an organic substance), are not generally included within the scope of the plastic trade as it is known today, inasmuch as the industries utilizing these materials are considerably older and were already individually organized and developed prior to the advent of the newer plastics.

In order to meet a growing demand for information regarding the organic plastics, a circular, C411, has been prepared on this subject. The various organic plastics are discussed from the standpoint of the raw materials required, the chemical reactions involved, the various methods of processing, and the more important applications.

The synthetic resins are subdivided into several chemical types as follows: Phenolic-aldehydic, amino-aldehydic, vinyl, hydroxy-carboxylic, indene, organic-polysulphides, and miscellaneous. Some of the more familiar industrial products which are discussed in this section of the circular are "Bakelite," "Catalin," "Beetleware," "Vinylite," "Glyptal," "Thiokol," and "Duprene." Natural resins of animal, vegetable, and mineral sources are described. Typical

examples of the natural resins from these three sources include shellac, rosin, and asphalt, respectively. The chemistry of the cellulose esters and ethers and cellulose xanthate is outlined. "Celluloid," "Fiberloid," "Plastacele," "Tenite," and "Cellophane" are among the well known cellulose plastics discussed. The protein plastics considered include those prepared from casein (milk), blood albumin, and soy beans. Selected references are listed at the conclusion of the circular for the convenience of the reader who may be interested in further details regarding the manufacture, properties, and uses of organic plastics.

Copies of this circular are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

HIDING POWER OF PAINT.

A new board to be used in determining the hiding power of paint has been constructed. This consists of 36 white and 36 black, opaque, glass squares (2 inches square) laid together in checkerboard fashion. In order to get the top surface as level and uniform as possible, the squares were laid face down on a smooth, level surface (a piece of plate glass), and cement placed on their backs, thus building up from the back of the squares. A layer of neat cement was laid next to the squares, followed by a 3 to 1 mixture of sand-portland cement mortar. After waiting for two weeks for the cement to harden, the board was turned over, face side up, resulting in an excellent surface for brushing and applying paint. The area of the board is 4 square feet. The reflection factor of the white squares is 80 per cent. and of the black squares less than 1 per cent. The squares of glass are so laid that no edge or frame hampers the brush in sweeping across the entire surface of the board. The board may be used for either dry or wet hiding power.

The board is mounted in a hood illuminated by light diffusely transmitted through a daylight filter from a bank of photoflood lamps. The color temperature of the transmitted light is about 6,500° K. The illumination on the surface of the hiding-power board is 165 foot-candles; it is uniform over

the board, and it is mostly incident normal to the board or nearly normal. For those not equipped with a standard illuminant, viewing the board at a north window illuminated by a fairly light overcast sky will approximate the conditions. The direction of view is about 45° from the normal to the board.

SAFETY GLASS FOR GLAZING AUTOMOBILES.

The motor vehicle laws of several States require the use of safety glass conforming to the safety code recently adopted by the American Standards Association as American Tentative Standard Z26.1–1935.

The preparation of this safety code was sponsored by the Bureau and the National Bureau of Casualty and Surety Underwriters, and the tests include those for resistance to impact as determined by dropping: (1) a half pound steel ball, (2) a 7-ounce steel dart, (3) an 11-pound shot bag onto the glass from appropriate heights. In some cases the glass does not break and in other cases the type of fracture is the deciding factor. The glass was also tested for discoloration by exposing it to radiations from a quartz-mercury lamp and to the effect of moisture at 125° F. and 70 per cent. relative humidity. Glass for use as windshields was also tested for distortion by projecting the image of a straight line through the glass.

Several of the States asked the Bureau to make the tests for them. Nine domestic manufacturers of safety glass furnished 768 samples representing 32 types of glass which were then tested. There are three kinds of safety glass covered by the code, namely, laminated glass, wire glass (meshed wire) and the so-called heat-treated or case-hardened glass.

Reports of the tests were sent to motor vehicle regulatory officials of 22 States who requested them and will be used (when applicable) by those officials for establishing lists of safety glass acceptable in those States. Reports may also be obtained by any other State official desiring them.

FORCEPS FOR HANDLING RADIOACTIVE SUBSTANCES.

Light weight forceps have been designed and constructed at the Bureau which permit the operator to pick up radioactive samples, with the nearest part of the hand at least 20 cm. from the sample. These forceps automatically grip and hold objects in the jaws by means of spring pressure. Pressure is applied by the operator only when it is desired to open the jaws to pick up or release a specimen. They may be made in right and left hand models and the jaws may be formed to pick up tubes of radium a millimeter or less in diameter. It has been found that several different forceps with specially formed jaws are desirable if tubes of a wide range of diameters are to be handled.

VELOCITY OF FLAMES FROM GAS BURNERS.

The velocity with which flame is propagated in the combustible mixture issuing from the ports of a gas burner is one of the important factors which must be taken into account in any study of the performance and design of such burners, and of the utilization of the various fuel gases which are supplied to the public. A better understanding of flames and the speed with which they travel should lead to more effective, satisfactory and safe utilization of these gases.

Notwithstanding the years of study which have been devoted to flames and the mechanism of combustion, many workers using different methods still obtain different results which cannot be readily correlated or even compared on a common basis. This comes largely from the profusion of variables, some of them controllable, some not, and some probably unknown.

A study of the burner method of measuring flame velocities has been made at the Bureau, one object of the study being to separate, one from the other, the effects of changing the size of the burner port, of changing the rate of flow and composition of the combustible mixture, and of using measurements of different quantities and of different parts of the flame surface in computing the result. The apparatus and procedure are described in detail in RP900 in the July number of the *Journal of Research*.

The relationships developed between the size and shape of the flame surface as related to the diameter of the burner port and the composition of the mixture lead to the following conclusions:

(1) The most consistent results by the burner method are obtainable when using mixtures in nearly theoretical proportions in laminar flow from burner ports over 4 mm. in diameter, and then computing the result by multiplying the average velocity of flow of the mixture by the sine of the angle between the axis and side of the flame surface at a point 0.7 the radius of the port from the axis.

(2) While the range of operations within which the burner method is capable of yielding reliable results is restricted considerably by the effects of the several experimental variables on the shape of the flame surface, yet it appears to be as simple and useful as any of the methods available at present. A knowledge of its limitations is essential, however, if erroneous conclusions from results obtained with it are to be avoided.

A derivation of the shape of the flame surface by the application consecutively of a series of facts, and assumptions based on facts, lead to the conclusions that: (1) the flame speed at the extreme tip may be several times as great as it is over the rest of the flame surface; (2) the flare at the base of the flame surface where it overhangs the burner port is better explained in terms of the magnitude and direction of flow of the gas mixture than by a cooling effect of the port; (3) the gap between the flame and the metal port is considered to represent the distance required by the temperature gradient from the port to the flame front, and, in consequence, represents approximately the thickness of a gradient layer underlying the entire flame surface, without specific assumptions regarding the mechanism involved in the propagation of the flame; (4) The curvature of the surface of most flames changes from concave at the base to convex at a considerable distance from the extreme tip. This is thought to be caused more by a modification of the originally parabolic distribution of the velocity of flow of the gas mixture, after leaving the burner tube than by an increase in the flame speed resulting from a higher initial temperature of the combustible mixture.

ANALYSIS OF THE FIRST SPECTRUM OF VANADIUM.

Light from vanadium vapor excited to luminosity in an electric arc, when dispersed by a spectrograph is seen to consist of several thousand different monochromatic radiations. These radiations resulting from atomic energy changes are controlled by quantum theory and their interpretation, therefore, leads to deductions of atomic structure. All available data for 2,500 such radiations or spectral lines are compiled and analysed in RP906 in the July number of the *Journal of Research*. The wave-lengths range from ultraviolet (2,000 A) to infrared (12,000 A) and these fundamental data are supplemented by estimated relative intensities, temperature excitation stages, line splitting in magnetic fields (Zeeman effect), and absorption observations.

The analysis resulted in the identification of 60 doublet, 60 quartet, and 28 sextet terms (atomic energy states) which combine to explain 2,186 observed lines. The principal features of the first spectrum of vanadium are thus disclosed, the descriptive material is satisfactorily interpreted and correlated, and all is found to be in excellent agreement with the modern theory of complex spectra. The spectrum is accounted for by 5 (valence) electrons out of a total of 23 in the extra-nuclear structure of vanadium atoms, and most of the identified spectral terms have been definitely assigned to particular electron configurations. The most intense line of the entire spectrum is a violet one (4,379.24 A) due to simple transition of a single electron between quantum states represented by azimuthal quantum numbers zero and one. This line is extremely sensitive for spectroscopic detection of vanadium.

Series of spectral terms have been recognized, which, when extrapolated to the limit, lead to a value of the energy (ionization potential) required to remove an outer electron from a vanadium atom. Thus it is found that a free electron falling through a potential difference of 6.71 volts acquires just sufficient energy to knock off an electron from a vandium atom upon collision.

MEASUREMENT OF ELECTROLYTIC RESISTANCE.

In certain precise measurements of electrolytic resistance the platinum electrodes are necessarily very thin so that the drop of potential in them cannot be neglected. In RP905 in the July number of the *Journal of Research*, formulas are obtained for this drop in the case of two types of cylindrical cells, one in which the current is axial, the other partly axial and partly radial. The potential admits of accurate evaluation in the first case and the results obtained confirm the method outlined for the treatment of a general shape of cell.

RAPID ELECTRODEPOSITION OF IRON.

The production of ductile electrolytic iron has been studied at the Bureau, using high current densities (from 100 to 300 amperes per square foot). It was found that the favorable range was extremely sensitive to the temperature, concentration of iron salts, and the acidity of the solution. The addition of calcium chloride and boric acid had no effect on the plating range. The study was made by measuring the tensile strength, elongation, and Brinell hardness of suitably prepared specimens.

DETERMINATION OF THE BRINELL NUMBER OF METALS.

The Brinell test has been used extensively in testing metals ever since its introduction by the Swedish engineer, J. A. Brinell, in 1900. He proposed to measure a property of the metal (which has been called the Brinell hardness number or Brinell number) in the following manner: A 10 mm. steel ball is forced into a specimen under a standard load, usually 3,000 kg., applied normal to the surface of the specimen, and the diameter of the identation after removing the load is measured. From the diameter, the surface of the portion of the indenting ball that was embedded in the specimen is calculated, the ratio of the load to the surface being defined as the Brinell number of the metal. By definition the Brinell number is equal to the average axial stress with which the material will resist further indentation.

Brinell and others after him found that the Brinell number of an iron or iron alloy specimen was roughly proportional to

the tensile strength of that alloy. The Brinell number was also found to be a convenient indicator of variations in work-hardening and other metallurgical properties in a given type of metal.

These valuable characteristics, together with the fact that the Brinell test is simple to make and, in contrast to the tensile test, does not involve destruction of the material, have led to its wide-spread adoption as an inspection test in the automotive industries and as a research tool in metallurgical laboratories.

With the increased use of the Brinell test it became important to reduce to a minimum the variations in determinations of the Brinell numbers of a given metal as measured by different observers at different locations. An extensive literature has consequently grown up which treats of the effect on the value of the Brinell number of different variables entering into the test. In the July number of the *Journal of Research* (RP903) there are brought together some of the results of such investigations and these have been supplemented by results of new tests and new analyses wherever the existing data seemed deficient. Discrepancies due to variations in test procedure, in the shape of the specimen, in the method of indenting, and variations in the type of ball used are considered at length. The conclusions of this review of old and new data on the Brinell test are summarized in recommendations for a test procedure designed to lead to greater concordance in the Brinell numbers obtained by different observers testing a specimen of given material.

A Unique Tin Plate.—T. W. LIPPERT in *Iron Age*, Vol. 138, No. 1, states that the Pittsburgh Crucible Steel Co. has erected a sizeable plant at Midland, Pa., for the first commercial venture for the electro-deposition of tin onto strip steel. Confidence of large scale operation is shown of this process which has emerged from the laboratory. Five thousand pound coils of 20-inch cold-reduced strip will be fastened end to end and will be fed through tin plating baths and processing equipment. Ingots will be reduced to blooms and sent to a hot mill for reduction to strips for cold rolling on a 26-inch reversing mill. At the discharge end, it is stated, there will be a continuous roll of tinned strip steel having a uniform coating, a dense and lustrous tin structure with almost complete freedom from pores, an elimination of pickling embrittlement and concomitant improvement in deep drawing qualities, a surface layer of pure tin and therefore improved corrosion resistance, and the avoidance of grease lines, mottle and isolated $FeSn_2$ crystallites. Much importance is attached to the resulting product especially in view of its many possible uses.

R. H. O.

Studying Electron Footprints.—(*Electric Journal*, 33, No. 6.) Electrons are much too small to see, but they can be induced to leave " footprints " plainly visible. A new tube has been developed for use in laboratories and classrooms where it is desired to display the behavior of electrons under different conditions. The tube has three electrodes—filament, grid, and plate. The plate is coated with salts that produce a fluorescent effect when bombarded by electrons. With a negative grid voltage, for instance, strips of light are made across the face of the plate, the width of the strips being adjustable by altering the grid voltage. The effect of an external magnetic influence on the electron pattern, and many other interesting effects can be similarly studied.

R. H. O.

THE FRANKLIN INSTITUTE

LIBRARY NOTES.

RECENT ADDITIONS.

Aircraft Year Book for 1936. Eighteenth Annual Edition; Howard Mingos, Editor. 1936.

ALFORD, L. P. Henry Laurence Gantt, Leader in Industry. 1934.

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CLOUZOT, HENRI. Histoire de la Manufacture de Jouy et de la Toile Imprimée au xviii^{e} Siècle. Two volumes text and plates. 1928.

CUGLE, CHARLES. Cugle's Practical Navigation. New and Enlarged Edition. 1936.

DURAND, WILLIAM FREDERICK. Robert Henry Thurston, a Biography. The Record of a Life of Achievement as Engineer, Educator, and Author. 1929.

ENGELDER, CARL J., TOBIAS H. DUNKELBERGER, AND WILLIAM J. SCHILLER. Semi-Micro Qualitative Analysis. 1936.

Engineering News-Record. Construction Costs. Ninth Edition. 1936.

Great Britain. Department of Scientific and Industrial Research. Review of Oxidation and Scaling of the Heated Solid Metals. 1935.

HAAS, ARTHUR. Atomtheorie. Dritte, völlig umgearbeitete und wesentlich vermehrte Auflage. 1936.

HENGLEIN, F. A. Grundriss der chemischen Technik. Ein Lehrbuch für Studierende der Chemie und des Ingenieurfaches, ein Übersichtsbuch für Chemiker und Ingenieure im Beruf. 1936.

MANNING, G. P. Reinforced Concrete Design. Second Edition. 1936.

MELLOR, J. W. A Comprehensive Treatise on Inorganic and Theoretical Chemistry. Volume 15, 1936.

Metals and Alloys. Cumulative Index of Metallurgical Abstracts Published in Volumes 1 and 2 and 6 of Metals and Alloys, July 1929 to December 1931, and January 1935 to December 1935.

Metropolitan Museum of Art. Benjamin Franklin and his Circle. A Catalogue of an Exhibition. 1936.

New Century Dictionary of the English Language. Edited by H. G. Emery and K. G. Brewster. Two volumes. 1936.

NOLAN, J. BENNETT. General Benjamin Franklin. The Military Career of a Philosopher. 1936.

SMITH, ALBERT W. John Edson Sweet. A Story of Achievement in Engineering and of Influence upon Men. 1925.

SNYDER, THOMAS ELLIOTT. Our Enemy the Termite. 1935.

South Kensington Science Museum. Handbook of the Collections Illustrating Marine Engines. By G. L. Overton. History and Development. 1935.

TAUBER, HENRY. Experimental Enzyme Chemistry. 1936.

WEBB, T. J. Elementary Principles in Physical Chemistry. 1936.

BOOK REVIEWS.

ATOMTHEORIE, von Arthur Haas, Dr. Phil., Professor für Physik an der Universität in Wien. 292 pages, tables, illustrations, 16.5 × 24.5 cms. Berlin und Leipzig, Walter de Gruyter & Co., 1936. Price 8.50 marks.

This is the third edition of a very interesting exposition of the fundamentals and high lights of atomic theory. The book is an outgrowth from lectures by the author, held at the University of Vienna. This edition, following those of 1924 and 1929, is thoroughly modern in method of presentation, and up to date in content.

There are seven chapters which conveniently divide the subject into a logical sequence. The theory of electrons is first considered and then the author proceeds through the fundamentals of atom mechanics including the theories of Broglie and Schrödinger, etc., spectroscopy, the structure of the atom envelope, and the effect or action between material and light. Everything usually considered under the heading of atomic theory is covered so that the student can become completely informed.

The book is outstanding not only for its clarity but especially for its avoidance of the use of higher mathematics. This brings it within the realm, in addition to physicists, of those less qualified.

R. H. OPPERMANN.

CUGLE'S PRACTICAL NAVIGATION, by Charles H. Cugle. 546 pages, folded plates, 16 × 24 cms. New York, E. P. Dutton & Co., Inc. 1936. Price $7.00.

A practical book for the practical man. It gives all the rules and problems in navigation used in every day work at sea. It is a consolidation of the author's previous works, "Simple Rules and Problems in Navigation" and "Practical Advanced Navigation."

The author starts with the international and inland rules of the road and then treats on navigation definitions and accessories, and deck officer's general information. Definitions used in ship stability, questions and answers in steel hull construction, arithmetic of navigation, and all the latest methods of determining a ship's position at sea are treated on. There are also printed Extracts from the

American Nautical Almanac for 1936, which cover all of the problems in the book, and as the methods used in printing the Almanacs are similar in each succeeding year, it can be assured that by mastering the problems for this year, there will be no difficulty in working problems for any succeeding year.

Because of its clarity and completness, this book should have wide appeal and be invaluable to the student as well as the practical marriner.

R. H. OPPERMANN.

DER AUFBAU DER ZWEISTOFFLEGIERUNGEN, EINE KRITISCHE ZUSAMMENFASSUNG, von Dr. phil. habil. M. Hansen. 1100 pages, illustrations, 16 × 24 cms. Berlin, Julius Springer, 1936. Price 87 RM.

As the title indicates this book contains valuable information on the combination of two metals into an alloy. It will be immediately recognized by those whose interest lies in this direction that a complete work of this nature will fill a very definite need. The information presented is the result of many years of evolution, the literature of which has been widely scattered. Its compilation into this volume must have been a tremendous task.

Examination of the contents reveals that all metals from Ag to W are covered in sequence and in conciseness with a very helpful and illuminating condition-curve for various temperatures. For example Ag-Al is first treated on. This covers about 4½ pages and reviews and gives results of various investigators, references to whose work are given at the end of the treatment. Then Ag-As is taken up in like manner and so on to and including W-Zn and W-Zr. By means of this work it is possible to see at a glance which alloy in a two metal system can become useful and which is of only technical interest; also a progress picture is presented by which the quality of the alloy at various stages may be determined. The subject index in the back of the book provides a convenient cross reference when considering the contents in the front.

As a compilation of the work of many years by a great number of investigators throughout the world, as a summarization of this work, as a ready reference work, this book is an outstanding accomplishment that will find much use by researchers and metallurgical technologists.

R. H. OPPERMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 555, Air Flow Around Finned Cylinders, by M. J. Brevoort and Vern G. Rollin. 13 pages, tables, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

A study was made to determine the air-flow characteristics around finned cylinders. Air-flow distribution is given for a smooth cylinder, for a finned cylinder having several fin spacings and fin widths, and for a cylinder with several types of baffle with various entrance and exit shapes.

The results of these tests show: That flow characteristics around a cylinder are not so critical to changes in fin width as they are to fin spacing; that the entrance of the baffle has a marked influence on its efficiency; that properly designed baffles increase the air flow over the rear of the cylinder; and that these tests check those of heat-transfer tests in the choice of the best baffle.

Report No. 556, Further Studies of Flame Movement and Pressure Development in an Engine Cylinder, by Charles F. Marvin, Jr., Armistead Wharton, and Carl H. Roeder. 13 pages, illustrations, 23 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

The investigation described in this report was carried out at the National Bureau of Standards at the request and with the financial assistance of the National Advisory Committee for Aeronautics. Stroboscopic apparatus, previously described, for observing flame movement through a large number of small windows distributed over the head of a spark-ignition engine was used in following flame spread with combustion chambers of different shapes at two engine speeds and for a variety of spark-plug locations including single and twin ignition. The principal factors influencing flame movement in the engine are discussed, and the lack of reliable information regarding their separate effects upon the structure of the flame and its speed of propagation is emphasized.

R.

ELEMENTARY PRINCIPLES IN PHYSICAL CHEMISTRY, WITH SPECIAL REFERENCE TO THE STATE OF EQUILIBRIUM IN A CHEMICAL REACTION AND TO THE RATE OF ATTAINMENT OF THE STATE OF EQUILIBRIUM, by T. J. Webb. 344 pages, tables, 15.5 × 23 cms. New York and London, D. Appleton-Century Company, Inc., 1936. Price $4.00.

The author explains that this textbook has been written as an introduction to the subject of physical chemistry primarily for juniors and seniors in college. It has been assumed that such students would have had preliminary training in the descriptive phases of chemistry, in general physics and in the calculus. Having these prerequisites there should be little difficulty in following through this book.

After a short introduction that is in the nature of a reminder of the basic laws, the book is divided into eight parts, as follows: the kinetic theory which includes a complete treatment on Maxwell's Distribution Law, a remarkable exposition of the first and second laws of thermodynamics between which is inserted a part on chemical equilibrium with examples and a description of the mechanism of electrolytic conductance and ionic equilibria, equilibrium between phases, the thermodynamic properties of strong electrolytes wherein there is given inclusive consideration of the theory of Debye and Hückel, the rates of chemical reactions, and the quantum theory. Therefore the structure of the book shows care in selection so as to accomplish its aims.

The method of presentation is one of continuous progression even, at times, to the point of vigorousness. In these instances one may have the feeling of a need for rest or a breathing spell, particularly when it becomes necessary to use the tool of mathematics to a greater extent. This is not a drawback. Rather is it a signal to proceed slower and become more adept at using the tool and at the same time obtain a thorough knowledge of derivations that are sufficiently general as not to need revisions when the advanced stages of the subject are reached.

On the other hand, the author throughout the book makes many notations very valuable in the way of short cuts for the purpose of recollection. In this respect he seems to have an insight into the mind of the student, to determine just what is necessary to recall a law and formula. This is set down in an impressive way.

The preface contains a list of sections of the book that may be omitted completely without serious loss of continuity, for those without a good working knowledge of the calculus. This includes the chapter on the quantum theory which can be regarded as a review of the leading elementary ideas. The choice of the illustrative applications here was made with the subject matter of the rest of the book in mind.

The book can be recommended to all those in need of a good treatment of the subject.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

Astronomy for the Layman, by Frank Reh. 308 pages, tables, plates, 14 × 21 cms. New York and London, D. Appleton-Century Company, Inc., 1936. Price $3.00.

Frontiers of Science, by Carl Trueblood Chase, Ph.D. 352 pages, plates, 14 × 22 cms. New York, D. Van Nostrand Company, Inc., 1936. Price $3.75.

Physical Chemistry, by Frank H. MacDougall, M.A., 721 pages, tables, illustrations, 14.5 × 22.5 cms. New York, The Macmillan Company, 1936. Price $4.00.

Franco Rasetti, Il Nucleo Atomico, 229 pages, tables, illustrations, 16 × 24 cms. Bologna, Nicola Zanichelli, 1936.

An Elementary Survey of Modern Physics, by Gordon Ferrie Hull. 457 pages, tables, 14.4 × 22.4 cms. New York, The Macmillan Company, 1936. Price $4.50.

A Textbook of Physics for Students of Science and Engineering, by Charles A. Culver. 816 pages, tables, illustrations, 15 × 22 cms. New York, The Macmillan Company, 1936. Price $4.00.

Actualites Scientifiques et Industrielles, 271, VIII, Evolution de l'Atmosphere Circulation Organique Epoques Glaciaires, par V. A. Kostitzin, 44 pages, price 12 francs. 277, IX, Verifications Experimentales de la Theorie Mathematique de la Lutte pour la Vie, par Dr. G. F. Gause, 61 pages, price 18 francs. 295, Lecons de Zoologie, par M. Prenant, Protozoaires Infusoires Cilies, 77 pages, price 15 francs. 296, Lecons de Zoologie, par M. Prenant, Protozoaires Flagelles, 55 pages, price 12 francs. 300, V, Determinisme et Realisation dans le Devenir du Sexe, par Vera Dantchakoff, 74 pages, price 18 francs. 303, V, Les Mouvements Morphogenetiques dans le Developpement des Vertebres, par Pierre Hatt, 57 pages, price 12 francs. 6 pamphlets, 16.5 × 25.5 cms. Paris, Hermann & Cie, 1935.

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Canada Dominion Bureau of Statistics, Report on the Fur Farms of Canada, 1934. 169 pages, tables, 16.5 × 25 cms. Ottawa, King's Printer, 1936. Price 25 cents. The Pulp and Paper Industry, 1934. 121 pages, tables, 16.5 × 25 cms. Ottawa, King's Printer, 1936. Price 25 cents. Advance Report on the Fisheries of New Brunswick, 1935. 9 pages, 22 × 28 cms. Ottawa, King's Printer, 1936.

Canada Department of Mines, Natural Bonded Moulding Sands of Canada, by Correll H. Freeman, 144 pages, tables, plates, 16.5 × 25 cms. Ottawa, King's Printer, 1936. Price 25 cents.

Bell Telephone Laboratories, Monographs: B-918, Applications of X-Rays to Industrial Problems, by J. R. Townsend and L. E. Abbott, 18 pages, illustrations. B-919, Diffuse Rings Produced by Electron Scattering, by L. H. Germer, 7 pages, illustrations. B-920, An Extension of Operational Calculus, by John R. Carson, 3 pages. B-921, Hyper-Frequency Wave Guides—Mathematical Theory, by John R. Carson, Sallie P. Mead and S. A. Schelkunoff, 24 pages. B-922, Resistance Fluctuations in Carbon Microphones and Other Granular Resistances, by C. J. Christensen and G. L. Pearson, 27 pages, illustrations. B-923, Some Contemporary Advances in Physics—XXX The Theory of Magnetism, by Karl K. Darrow, 24 pages, illustrations. B-924, Proportioning of Shielded Circuits, by E. I. Green, F. A. Leibe and H. E. Curtis, 36 pages, illustrations. B-925, A Magneto-Elastic Source of Noise in Steel Telephone Wires, by W. O. Pennell and H. P. Lawther, 6 pages. B-926, Reliability of Short-Wave Radio Telephone Circuits, by R. K. Potter and A. C. Peterson, Jr., 16 pages, illustrations. B-927, Hyper-Frequency Wave Guides—General Considerations, by G. C. Southworth, 26 pages, illustrations. 10 pamphlets, 15.5 × 23 cms. New York, Bell Laboratories, 1936.

National Advisory Committee for Aeronautics, Technical Notes: No. 569, Boosted Performance of a Compression-Ignition Engine with a Displacer Piston, by Charles S. Moore and Hampton H. Foster, 9 pages, figures. No. 570, Effect of Changes in Tail Arrangement Upon the Spinning of a Low-Wing Monoplane Model, by C. H. Zimmerman, 8 pages, figures. No. 571, A Method of Estimating the Aerodynamic Effects of Ordinary and Split Flaps of Airfoils Similar to the Clark Y, by H. A. Pearson, 17 pages, figures. 3 pamphlets, 20 × 26 cms. Washington, Committee, 1936.

CURRENT TOPICS.

Work and Fatigue.—H. W. HAGGARD. (*Mechanical Engineering*, Vol. 58, No. 5.) The author shows some of the physiological errors into which the words "work" and "fatigue" may lead. The terms "work" and "rest" are quite erroneously taken to express entirely opposite conditions. In reality they are not opposite; they are the same conditions, varying only in degree. The human body, as long as it is alive is never at rest. Even in the most extreme muscular relaxation, the muscles are active and the vital functions of all the organs are carried on continuously. Often the energy directly expended for the performance of light manual tasks is considerably less than that expended by the body during the state of so-called complete rest. The average man sitting at rest, performing no task expends energy at the rate of about 100 calories per hour. The same man performing an occupation such as typing, increases the expenditure to about 140 calories per hour. The energy directly expended is thus only 40 per cent. of that expended while sitting at rest. The development of fatigue, of course, results from strenuous exertions. It is a demonstrable physiological condition arising from the fact that energy is expended by the muscles in excess of the rate of the reparative processes. Finally the condition is relieved by rest. Many industrial occupations, however, are of such a nature that they result in progressive development of fatigue without necessarily being the result of energy directly expended in work. Tests were made with ingenious apparatus on muscular efficiency at hourly intervals throughout the day on a number of subjects and it was found that even for manual operations too light to produce true fatigue the efficiency diminishes. It was shown in factory tests, that the taking of extra meals during the day produced less fatigue and accomplished more work.

R. H. O.

Synthetic Atmospheres Studied.—(*Heating and Ventilating*, Vol. XXXIII, No. 6.) A twelve-year study of synthetic gases in relation to animal life shows that some artificial atmospheres can be created which challenge the superiority of the atmosphere of the world for maintaining life, according to Dr. J. Willard Hershey in a paper presented before the American Society of Chemical Engineers

recently. A wide range of synthetic atmospheres in the confinement of animals have been used. Helium and oxygen apparently are better than normal atmospheres. The best percentage mixture seems to be 50 per cent. each. Little is known of the very large field of synthetic atmospheres. In the field of practical application there is a wide range of values and uses. In deep sea diving, mines, and submarines, foul air is encountered and there is often a lack of sufficient pure air to sustain life. Aviators may carry a supply of prepared atmosphere with them when flying in the substratosphere. According to Dr. Hershey the widest field probably will lie in the pathological applications.

R. H. O.

The Largest 3,000-r.p.m. Turbo-Generator.—(*Engineering Progress*, Vol. XVII, No. 5.) This single shaft machine, a product of Siemens-Schuckertwerke, was recently started into service in the central power station at Schelle near Antwerp, Belgium. The difficulties in design and in the production of the large rotor forgings from high grade steel alloys of the requisite high strength, purity, homogeneity, and toughness were overcome in close coöperation between the designer and the metallurgist. The machine has a capacity of 80,000 KVA and 10,500 volts. The rotor is in three parts, comprising the central section, or active rotor proper, which takes the form of a hollow body about 16.4 ft. long and 3.3 ft. in diameter, and two flanged-on stub shafts. This design rendered it possible thoroughly to forge the ingot and bring the material to a good condition of structure. It is made from a heat treated chrome-nickel steel. The windings are of copper and are accommodated in thirty-six slots. Coil ends are freely movable, protected from the effect of centrifugal force by caps of seamless forged non-magnetic special steel shrunk on their seats on the rotor body. The weight of the rotor is 42 tons. The stator frame is welded, while the end shields are from cast iron and the core end plates from steel. The stator with end shields weighs 114 tons. Special attention was given to effective dissipation of the heat from the copper and iron losses. Ventilation is supplied by turbo-blowers externally of the generator housing on the generator shaft. With completion and operation of this machine there is indication of not only a technical but also an economic advance as it is cheaper than the large-capacity generators running at 1500 r.p.m. built so far.

R. H. O.

Lightning Observatory.—(*Compressed Air Magazine*, Vol. 41, No. 4.) What is believed to be the only observatory for studying

natural lightning has been erected on the roof of the largest building of the Pittsfield, Mass., plant of the General Electric Co. This structure is equipped with a periscope and with a motor-operated, high-speed camera, both of which are designed to cover the entire horizon. There, research engineers under the direction of K. B. McEachron expect to obtain information of value to them in their work of improving transmission and distribution equipment so that far flung networks may be still further safeguarded. The structure on the outside is coated with aluminum paint and on the inside with flat black. It has a diameter of 14 ft. and in the center of it is the periscope which is topped by an 8 inch sphere. Lightning flashing in any direction is reflected in the silvery surface of the roof and thence in the crystal sphere and is made visible through the eyepiece of the periscope by a mirror set at an angle of 45 deg. in its dark-walled tube. The instrument has an effective range of 20 miles. Directly beneath the periscope platform is the camera, which is said to be one of the most unusual apparatus of its kind ever built. It is provided with a moving strip of film and with twelve lenses that make it possible to record all lightning discharges occurring anywhere within its 360 deg. reach. As the instrument is exposed to the weather when in use, it is necessary to protect the lenses against rain. This is accomplished by surrounding it with a curtain of compressed air. The air is taken from the plant's main supply and is admitted into a perforated ring beneath which the camera is placed. The invisible curtain shuts out all but the most severe downpours.

R. H. O.

An Engine That Will Run On Anything That Will Burn.—(*Power*, Vol. 80, No. 5.) RUDOLF PAWLIKOWSKI began, twenty or more years ago, to develop the diesel engine along the lines envisaged initially by Dr. Diesel—burning coal dust or pulverized coal fed through a rotary valve from a hopper. Mr. Pawlikowski has now added a supplementary chamber in which the fuel is retained practically throughout the compression stroke. It is thus heated about 10 times as long and intensively as in a diesel, meanwhile mixing thoroughly with primary air. Fuel from the hopper of the "Rupamotor" is metered through cylindrical valves into the supplementary chamber. A main fuel valve seals the opening into this chamber and controls the primary air supply. Fuel is dropped through shortly after the beginning of the intake. Primary air whirls the charge and also cleans the seat of the fuel valve as it closes. The position of the chamber, just over the cylinder, is

such that this air is drawn in by the engine itself, just as is the fuel, making it possible to operate the engine without pressure on either air or fuel. There is, however, a controllable compressed air injection valve that admits auxiliary air as desired to scavenge the chamber and blow slow distilling fuel particles into the cylinder, although the chamber can normally be kept clean without this air. The engine will operate on a variety of fuels including coal, coke, potato peelings, sawdust, leaves, wood residues, hay, corn stalks, etc. Thermal efficiencies given for the stationary Rupamotor are 32 per cent. with anthracite, 30 per cent. with lignite. For the automotive Rupa efficiencies given are 22.5 per cent. with lignite having 4 to 6 per cent. ash, 22 per cent. with sawdust "flour," and 21 per cent. with forest and field residues that are finely ground.

R. H. O.

D.C. Transmission in France.—(*Electrical World*, Vol. 106, No. 19.) There is a d.c. line in France in regular operation transmitting up to 20,000 kw. at a voltage of 125,000 over a distance of 275 miles between the cities of Moutiers, in the Tarentaise Alps, and Lyon (population 539,581), in the valley of the Rhone. Forty-five miles of the line runs in cable and the remaining 230 miles is overhead construction. The line is about thirty years old and the original voltage was some 55,000. Power is generated near Moutiers by four groups of four generators each, all connected in series. The 230-mile overhead line passes through four sectionalizing stations and enters the receiving station on the outskirts of Lyon, where some of the power is taken by three sets of motors, also connected in series. These drive three sets of three phase generators. Here is where the cable installation originally began, extending to the end of the line in downtown Lyon. At this point five sets of motors, all in series, convert the rest of the d.c. power into mechanical power, which drives five d.c. generators for supplying the Lyon street railways. Power can be fed into the system at numerous points and apparently the system is quite flexible. The generating stations of the system are so designed that one or more groups of generators can be inserted into the closed circuit formed by the line. When this is done and the newly started generator attains proper voltage, the other generators assume a new voltage and the system is in effect a constant-current, variable voltage system. To accomplish this, current regulators are used.

R. H. O.

New Timken Steel—"Silmo." (*Heat Treating and Forging*, Vol. XXII, No. 5.) The Timken Steel and Tube Co. has just announced a new steel designated as "Silmo," especially designed for applications where an economical combination of high temperature, strength and oxidation resistance is required. In the temperature range of 1000–1200 deg. F. this steel has about twice the strength of carbon steel and its oxidation resistance compares favorably with that of 4–6 per cent. Cr-Mo steel. This new steel may be used to replace carbon steel where greater safety is required and as a substitute for the standard carbon molybdenum steel where oxidizing conditions are encountered. Indicated uses for it lie in the field of cracking furnace tubes, pipe still heater tubes, high pressure boiler and superheater tubes, and tubing used in the manufacture of air heating equipment.

R. H. O.

An Epoch in Communication.—In the year 1836 Samuel F. B. Morse gave the first demonstration of his electric telegraph to his colleagues at New York University. He gave the next demonstration outside of New York City before the membership of The Franklin Institute, in Philadelphia. It was on this occasion that the Committee on Science and the Arts of The Franklin Institute in reporting on the apparatus stated, in part:

> "The committee beg to state their high gratification with the exhibition of Prof. Morse's telegraph, and their hope that means may be given to him to subject it to the test of an actual experiment made between stations at a considerable distance from each other."

This was a century ago. Since then the world has witnessed tremendous strides of scientific development. The Franklin Institute has played its part in the promotion of science and the mechanic arts.

On June 11, 1936, communication hailed the approach of a new service—that of an ultra-short-wave radio circuit connecting New York and Philadelphia. In the centennial year of Morse's demonstrations the same two institutions witnessed again an epoch making development in communications. Chancellor Harry Woodburn Chase of New York University and Vice President W. Chattin Wetherill of The Franklin Institute by means of this ultra-short wave circuit, exchanged pictures and greetings. David Sarnoff, President of the Radio Corporation of America, stated to Guests at the New York end of the radio circuit, that the three meter band of radio wave-lengths provide almost unbelievable possibilities. Messages

can be sent not only in facsimile, but two pictures can be sent simultaneously and there can be added two automatic typewriter channels and a telegraph channel, all to operate in both directions at the same time.

Between the terminal stations at New York and Philadelphia, there are two repeater stations, one at New Brunswick and one at Arney's Mount, near Trenton N. J. Each repeater station is automatic in operation, and operates only when the circuit is being used. Each employs two different transmitting wave-lengths, one for each direction. The two terminal stations each use one sending wave, making a total of six wave-lengths, or frequencies, for the complete circuit. If it should be desired to extend the circuit beyond either terminal point the same six waves could be used in the same sequence and not cause interference because of the line-of-sight limitation to their range.

The transmitting and receiving apparatus embody the principle of "resonant lines," which was developed by RCA and eliminates crystal control and provides economical and efficient means of keeping in steady tune. The heart of the receiver is the "shoe button" or "acorn" tube and in the transmitters there are newly developed power tubes and other apparatus, all by RCA. Research work and development of improvements now under way promise much in furthering efficiency and simplification of design.

R. H. O.

What Wood Can Do—But Doesn't.—J. W. WATZEK, JR., retiring president of the National Lumber Manufacturers Association, took as the text of his address a quotation by Dr. C. M. A. Stine, vice-president of the DuPont Company, namely: "More attention is being given today to the chemistry of wood than to the chemistry of steel, and that wood to the chemists has literally become more valuable than gold." No doubt such a statement is based, partially at least, upon the fact that more than 5,000 different classes of articles are now made directly or indirectly from wood cellulose, and that they embrace some 200,000 items.

The matter of making ethyl alcohol from wood has been fairly well worked out. A cord of cheap coniferous wood will produce 20 gallons of alcohol at a cost not exceeding 20 cents a gallon. Mental gymnastics are oft times difficult to resist. Some one estimated that 800,000,000 cords of wood annually would be required to make sufficient alcohol to replace the gasoline now being consumed in a like period. Right now the entire output of pulp and paper in the United States requires only 13,000,000 cords of wood.

A more probable development may be that of the "producer gas" automobile. In such cases, the wood is converted directly into producer gas which serves as the motivating power for an internal combustion engine. France, Germany, Austria and Italy, all have made extensives tests on the use of wood gas motors. About five years ago test operations on a French machine were conducted in this country by the joint labors of the army, a lumber company and a sugar refining plant near San Francisco. Operatively the tests were successful, but the cheapness of gasoline in this country, in view of its superiorities over wood gas, have prevented its general introduction.

Valuing wood at $6 a cord of 2,000 pounds, and gasoline at 20 cents a gallon, it was found in one of the army tests that producer gas resulted in a cost of two-thirds of a cent per hour per horsepower as against a cost of two-and-one-half cents for gasoline. One of the disadvantages of the wood gas engine is that it takes several minutes to start the generation of the combustible gas. Another is the space required for wood storage and the plant. Also, gasoline produces more rapid acceleration and starts more easily than the wood gas. A cord of wood will propel a vehicle for about 325 miles.

C.

For the forthcoming Third World Power Conference there has been prepared for exhibition the completely electrified Rosedale Dairy Farm in Virginia. Through electrification the old farm has been transformed into a model dwelling, more comfortable than the average city home. The farmhouse proper will be air conditioned. It will have electric heaters, electric churns, sunray lamps, vacuum cleaners, washing machines, dishwasher, electric stove, etc. Electricity will do most of the routine chores on the farm, such as wood chopping, feed grinding, milking and hay hoisting. Such luxuries of city life as electric vibrators, hair dryers, water heaters, are included as farm equipment. Night work on the farm is done by light of powerful flood lamps. Cows are milked electrically and instead of the conventional screens, the dairy house is equipped with electrically charged screens which electrocute the flies. To keep the bull and other live stock in bounds, a single wire fence with a light electric charge has been installed. The Virginia Public Service Co. is furnishing the power.

R. H. O.

The Translucency of Sweetness.—As an example of coöperative success the Mellon Institute tells how an incumbent of the Sugar Institute, Inc. fellowship developed a commercial method for the

production of sucrose octaäcetate and the applications of this compound in paper technology were then studied by the Strathmore fellowship which found it imparted to certain papers a mellow translucency. Such a use would be valuable in paper stock where strength, pleasing texture, excellent printing properties, and translucency are important considerations.

C.

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